

CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS

October 2025

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate value to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 5 questions on 3 pages.

Marks

Q. No	Time: 3 hours	Value	Earned												
1.	Answer all of the following: a) Right Ascension system is an inertial reference system. Explain why it is an inertial system and discuss how its curvilinear coordinates are measured (including their units and zero points of the coordinates). b) Describe the origin and coordinate axes of the coordinate system that is commonly used for satellite positioning at each specific instant of time the satellite is observed and explain four of the important parameters needed to convert coordinates in this coordinate system into geocentric coordinate system. c) Explain the differences between the Local Astronomic coordinate system and the Horizon coordinate system regarding their origins, linear coordinate axes, fundamental planes, and the observables in each. d) Explain the important differences between UT1 and UTC (stating their full names and the main properties distinguishing them).	5 8 8 4													
2.	<table><tr><td colspan="3">The map coordinates of points R and S in a stereographic double projection and the UTM projection are as follows:</td></tr><tr><td></td><td>Stereographic double Projection</td><td>UTM projection</td></tr><tr><td>Point R:</td><td>Northing = 7,528,656.555 m, Easting = 2,497,692.180 m Latitude: 46° 45' 28.0998" N, Longitude : 66° 31' 48.7559" W Grid Convergence: -0° 01' 19.06" (T-t) Correction : 0.04"</td><td>Northing = 5,181,212.674 m, Easting = 688,610.493 m Latitude: 46° 45' 28.0998" N, Longitude: 66° 31' 48.7559" W</td></tr><tr><td>Point S:</td><td>Northing = 7,519,393.255 m, Easting = 2,498,724.831 m</td><td>Northing = 5,171,985.701 m, Easting = 689,937.055 m</td></tr></table> a) Calculate the plane bearings of line RS in the stereographic double projection and UTM projection. Explain two important reasons why the two calculated plane bearings are different. b) Calculate (independently) the geodetic bearings of line RS from the plane bearings computed for the stereographic double projection and the UTM projection (assuming the mean radius of the earth in the mapping region, $R_m = 6,382,129.599$ m). Explain one important reason why the two calculated geodetic bearings are different. c) Determine the scale factor range (to 6 decimal places) and the number of projection zones for a large-scale cadastral mapping of a region (with 360 km East-West extent) so that the scaling accuracy ratio remains within 1/10,000. Assume the mean radius of the earth in the region is 6.371 km and a modified Transverse Mercator will be used.	The map coordinates of points R and S in a stereographic double projection and the UTM projection are as follows:				Stereographic double Projection	UTM projection	Point R:	Northing = 7,528,656.555 m, Easting = 2,497,692.180 m Latitude: 46° 45' 28.0998" N, Longitude : 66° 31' 48.7559" W Grid Convergence: -0° 01' 19.06" (T-t) Correction : 0.04"	Northing = 5,181,212.674 m, Easting = 688,610.493 m Latitude: 46° 45' 28.0998" N, Longitude: 66° 31' 48.7559" W	Point S:	Northing = 7,519,393.255 m, Easting = 2,498,724.831 m	Northing = 5,171,985.701 m, Easting = 689,937.055 m	4 18 6	
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3.	The map projection equations relating map projection coordinates (x, y) with the corresponding geographic coordinates (ϕ, λ) can be given as $x = R\lambda \qquad y = R \sin \phi$ where R is the mean radius of the spherical earth. Calculate the area distortion factor and determine (showing all your derivations and substitutions of values) if this projection is equivalent.	7	
4.	Explain each of the following (giving clear explanation of each of the terms involved, and do not be tempted to state, for example, that one is ... and the other is not) : a) One main difference between datum transformation and coordinate transformation. b) One main difference between reference system and reference frame. c) Compare CGVD28 with CGVD2013 by explaining how they are realized, types of height system, reference surfaces for defining them, and simple models for converting height from one system to the other (clearly showing the relationship between each system and NAD83(CSRS)). d) The relationship (with typical examples) between normal sections on a surface and the principal directions on that surface. e) The relationship between a map projection and its map grid (describing their origins, units and axes involved).	2 3 6 3 6	
5.	Using two well labelled sketches only, illustrate the Transverse Mercator (TM) and the Mercator projections in the Northern hemisphere; provide one sketch for the TM projection and the other sketch for the Mercator projection. Each sketch must show the projections of the Equator, Central Meridian (CM), not less than four parallels, not less than four meridians and not less than four Tissot indicatrices each along CM and Equator with appropriate relationship between lines of the graticule clearly illustrated.	20	
		100	

Some potentially useful formulae are given as follows:

$$T-t = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - 500,000$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i .

$$\text{UTM average line scale factor, } \bar{k}_{UTM} = 0.9996 \left[1 + \frac{x_u^2}{6R_m^2} \left(1 + \frac{x_u^2}{36R_m^2} \right) \right];$$

$$\text{where } x_i = x_i^{UTM} - 500,000; \quad x_u^2 = x_1^2 + x_1x_2 + x_2^2$$

$$\text{UTM point scale factor, } k_{UTM} = k_0 \left[1 + \frac{\Delta x^2}{2R_m^2} \right], \text{ where } \Delta x = x^{UTM} - x_0$$

Grid convergence, $\gamma_B = \Delta\lambda \sin \phi \left[1 + \frac{\Delta\lambda^2 \cos^2 \phi}{3(20265)^2} \right]$; where $\Delta\lambda = (\lambda - \lambda_0)$ (in arc-seconds)

for any given longitude λ with central longitude at λ_0 .

$$\alpha = t + \gamma + (T - t) \quad X_f = X_G \times K_f + X_0; \quad Y_f = Y_G \times K_f$$

$$Sf = \frac{R_m}{R_m + H_m}$$

$$x = (N + h) \cos \phi \cos \lambda; \quad y = (N + h) \cos \phi \sin \lambda$$

$$z = \left[(1 - e^2) N + h \right] \sin \phi; \quad N = \frac{a}{\sqrt{(1 - e^2 \sin^2 \phi)}}$$

$$\Delta r^{LG} = R_3(\eta_0 \tan \phi_0) R_2(-\xi_0) R_1(\eta_0) \times \Delta r^{LA}$$

$$\Delta r^G = R_3(\pi - \lambda_0) R_2\left(\frac{\pi}{2} - \phi_0\right) P_2 \times \Delta r^{LG}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B^G = r^G + \Delta r^G$$

$$\alpha_{ij} = A_{ij} - \eta_i \tan \phi_i - (\xi_i \sin \alpha_{ij} - \eta_i \cos \alpha_{ij}) \cot Z_{ij}$$

Distortion Formulas:

$$\text{Given: } X = f(\phi, \lambda) \quad Y = g(\phi, \lambda)$$

$$E = f_\phi^2 + g_\phi^2 \quad F = f_\phi f_\lambda + g_\phi g_\lambda \quad G = f_\lambda^2 + g_\lambda^2$$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2}; \quad m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2 \cos^2 \phi}; \quad p = \frac{2(f_\phi f_\lambda + g_\phi g_\lambda)}{R^2 \cos \phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A'_p; \quad \textcolor{blue}{f}_\phi = \frac{\partial f(\phi, \lambda)}{\partial \phi}$$

$$\sin A'_p = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{(f_\lambda g_\phi - f_\phi g_\lambda)^2 + (f_\phi f_\lambda + g_\phi g_\lambda)^2}}$$

$$\text{Equivalency Condition: } f_\lambda g_\phi - f_\phi g_\lambda = \pm R^2 \cos \phi$$

$$\text{Cauchy-Riemann Equations: } f_\phi = -\frac{1}{\cos \phi} g_\lambda; \quad g_\phi = \frac{1}{\cos \phi} f_\lambda$$

$$\sin\left(\frac{\omega}{2}\right) = \frac{(a-b)}{(a+b)}$$