



|    |                                                                                                                                                                                                                                                                                                                                                     |     |  |
|----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|--|
| 4. | a) A 3TM zone (with False Easting of 304,800 m and the scale factor of central meridian of 0.99990) and a UTM zone have the same central meridian. Calculate the UTM coordinates of the point whose 3TM map coordinates are X = 274,800.000 m, Y = 5,500,000.000 m.                                                                                 | 5   |  |
|    | b) Determine the longitude coordinates (along the Equator) of the points where the scale factor distortion is minimal in UTM Zone 10N projection. What is that scale factor distortion?                                                                                                                                                             | 5   |  |
| 5. | On an UTM projection, point A is located on the eastern secant line while point B is located on a line where the scale factor is 1.0001 towards the eastern boundary. If the UTM plane bearing of line AB is 45°30'10.5", determine the grid bearing of the line (assuming the Gaussian mean radius of the earth in the region is 6,382,129.599 m). | 20  |  |
|    |                                                                                                                                                                                                                                                                                                                                                     | 100 |  |

Some potentially useful formulae are given as follows:

$$T-t = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where  $y_i = y_i^{UTM}$ ;  $x_i = x_i^{UTM} - x_0$ ;  $R_m$  is the Gaussian mean radius of the earth; and  $x_i^{UTM}$  and  $y_i^{UTM}$  are the UTM Easting and Northing coordinates respectively, for point  $i$ .

$$\text{UTM average line scale factor, } \bar{k}_{UTM} = k_0 \left[ 1 + \frac{x_u^2}{6R_m^2} \left( 1 + \frac{x_u^2}{36R_m^2} \right) \right];$$

$$\text{where } x_i = x_i^{UTM} - x_0; \quad x_u^2 = x_1^2 + x_1x_2 + x_2^2$$

$$\text{UTM point scale factor, } k_{UTM} = k_0 \left[ 1 + \frac{\Delta x^2}{2R_m^2} \right], \text{ where } \Delta x = x^{UTM} - x_0$$

$$k_{UTM} = k_0 \left[ 1 + \frac{L^2}{2} \cos^2 \phi \right]$$

$k_0$  is scale factor of Central Meridian and  $x_0$  is the False easting value (or 500 000 m)

$L = (\lambda - \lambda_0)$  (in radians) for a given longitude  $\lambda$ ; and  $\lambda_0$  is the longitude of the central meridian.

$$\text{Grid convergence, } \gamma = L \left( 1 + \frac{L^2}{3} (1 + 3\eta^2) \cos^2 \phi \right) \sin \phi$$

where  $\eta^2 = e'^2 \cos^2 \phi$ ;  $e'^2 = 0.006739496780$ ;  $L = (\lambda - \lambda_0)$  (in radians);  $\lambda_0$  is the longitude of the central meridian; and  $\phi$  is the latitude of the given point.

Geodetic bearing:  $\alpha = t + \gamma + (T - t)$

$$Sf = \frac{R_m}{R_m + H_m}$$

Transformation Formulas:

$$X_{(target)} = k_{0(target)} X_G + X_{0(target)}$$

$$Y_{(target)} = k_{0(target)} Y_G$$

$$X_G = \frac{[X_{(original)} - X_{0(original)}]}{k_{0(original)}}$$

$$Y_G = \frac{Y_{(original)}}{k_{0(original)}}$$

ITRF:

$$\mathbf{r}(t) = \mathbf{r}_0 + \dot{\mathbf{r}}(t - t_0)$$

where  $\mathbf{r}_0$  and  $\dot{\mathbf{r}}$  are the position and velocity respectively at  $t_0$ .

Distortion Formulas:

Given:  $X = f(\phi, \lambda) \quad Y = g(\phi, \lambda)$

$$E = f_\phi^2 + g_\phi^2 \quad F = f_\phi f_\lambda + g_\phi g_\lambda \quad G = f_\lambda^2 + g_\lambda^2$$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2}; \quad m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2 \cos^2 \phi}; \quad p = \frac{2(f_\phi f_\lambda + g_\phi g_\lambda)}{R^2 \cos \phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A_p'; \quad f_\phi = \frac{\partial f(\phi, \lambda)}{\partial \phi}$$

$$\sin A_p' = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{(f_\lambda g_\phi - f_\phi g_\lambda)^2 + (f_\phi f_\lambda + g_\phi g_\lambda)^2}}$$

Equivalency Condition:  $f_\lambda g_\phi - f_\phi g_\lambda = \pm R^2 \cos \phi$

Cauchy-Riemann Equations:  $f_\phi = -\frac{1}{\cos \phi} g_\lambda; \quad g_\phi = \frac{1}{\cos \phi} f_\lambda$