

CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS

March 2026

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate value to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 6 questions on 3 pages.

Marks

<u>Q. No</u>	<u>Time: 3 hours</u>	<u>Value</u>	<u>Earned</u>
1.	Answer the following: a) Explain one main difference between NAD83(CSRS) and ITRF (including stating their full names). b) Explain one main difference between CGVD28 and CGVD2013 (including stating their full names). c) Explain one important difference between UT1 and UTC (including stating their full names). d) Explain specifically one important difference between UTM and MTM projections (including stating the full name of MTM). e) Explain <i>spatial reference system</i> and its reference frame (including how they are defined). f) Explain <i>combined scale factor</i> and how it is used.	3 3 3 2 3 3	
2.	In a certain boundary retracement project in a province in Canada, the preliminary surveys consisted of two independent measurement schemes: one made using GPS, producing geocentric coordinates; and the other by conventional traversing based on UTM system. The independently estimated coordinate data are to be used to compute inverses for the azimuths and distances for the final lines to be cut. A series of astronomical observations were also taken at each of the points that had been established for comparison with the azimuths from these two methods. Answer the following: a) Describe the coordinate systems (GPS and UTM coordinate systems) involved in the two measurement schemes (defining their origins and directions of axes). b) Clearly explain how the computed inverses for the azimuths will differ from the corresponding astronomic azimuths (your explanation must demonstrate the understanding of the three azimuths involved with the indication of the directions of north in each case). c) Describe how the inverses for the distances in the two measurement schemes will compare with each other (stating the distance types and how to transform from one type to another). d) Assuming that total station's mark-to-mark distance and angle measurements are made at each of the network points and used directly with the corresponding astronomic azimuths to produce another set of three-dimensional coordinate data, describe the (total station) coordinate system most likely established by this process (defining the origin and directions of axes of the coordinate system). e) Discuss one important use of reference ellipsoid in each of the following coordinate systems: UTM coordinate system, GPS coordinate system, and total station coordinate system (including in your discussion one important disadvantage of each system).	7 5 3 4 6	

3.	On a UTM projection, point A is located on the eastern secant line while point B is located on a line where the scale factor is 1.0001 towards the eastern boundary. If the UTM plane bearing of line AB is 45°30'10.5", determine the grid bearing of the line (assuming the Gaussian mean radius of the earth in the region is 6,382,129.599 m).	20	
4.	Given the general scale factors along the meridians and the parallels for a map projection system as $1/\cos\phi$ and the Gaussian surface element as $f = \theta$ (where ϕ is the latitude), determine mathematically if the map projection is conformal or equivalent and show that both distortion properties cannot be satisfied at the same time (showing all of your mathematical expressions and substitutions of values into them for full marks).	5	
5.	Answer all of the following: a) Right Ascension system is an inertial reference system. Explain why it is an inertial system and discuss how its curvilinear coordinates are measured (including their units and zero points of the coordinates). b) Explain the differences between the Local Astronomic coordinate system and the Horizon coordinate system regarding their origins, linear coordinate axes, fundamental planes, and the observables in each.	5 8	
6.	Using two well labelled sketches only, illustrate the Transverse Mercator (TM) and the Mercator projections in the Northern hemisphere; provide one sketch for the TM projection and the other sketch for the Mercator projection. Each sketch must show the projections of the Equator, Central Meridian (CM), not less than four parallels, not less than four meridians and not less than four Tissot indicatrices each along CM and Equator with appropriate relationship between lines of the graticule clearly illustrated	20	
		100	

Some potentially useful formulae are given as follows:

$$T-t = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - 500,000$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i .

$$\text{UTM average line scale factor, } \bar{k}_{UTM} = 0.9996 \left[1 + \frac{x_u^2}{6R_m^2} \left(1 + \frac{x_u^2}{36R_m^2} \right) \right];$$

$$\text{where } x_i = x_i^{UTM} - 500,000; \quad x_u^2 = x_1^2 + x_1x_2 + x_2^2$$

$$\text{UTM point scale factor, } k_{UTM} = k_0 \left[1 + \frac{\Delta x^2}{2R_m^2} \right], \text{ where } \Delta x = x^{UTM} - x_0$$

$$\text{Grid convergence, } \gamma_B = \Delta\lambda \sin\phi \left[1 + \frac{\Delta\lambda^2 \cos^2\phi}{3(20265)^2} \right]; \text{ where } \Delta\lambda = (\lambda - \lambda_0) \text{ (in arc-seconds)}$$

for any given longitude λ with central longitude at λ_0 .

$$\alpha = t + \gamma + (T - t) \quad X_f = X_G \times K_f + X_0; \quad Y_f = Y_G \times K_f$$

$$Sf = \frac{R_m}{R_m + H_m}$$

$$x = (N + h) \cos \phi \cos \lambda; \quad y = (N + h) \cos \phi \sin \lambda$$

$$z = \left[(1 - e^2) N + h \right] \sin \phi; \quad N = \frac{a}{\sqrt{(1 - e^2 \sin^2 \phi)}}$$

$$\Delta r^{LG} = R_3(\eta_0 \tan \phi_0) R_2(-\xi_0) R_1(\eta_0) \times \Delta r^{LA}$$

$$\Delta r^G = R_3(\pi - \lambda_0) R_2\left(\frac{\pi}{2} - \phi_0\right) P_2 \times \Delta r^{LG}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B^G = r^G + \Delta r^G$$

$$\alpha_{ij} = A_{ij} - \eta_i \tan \phi_i - (\xi_i \sin \alpha_{ij} - \eta_i \cos \alpha_{ij}) \cot Z_{ij}$$

Distortion Formulas:

$$\text{Given: } X = f(\phi, \lambda) \quad Y = g(\phi, \lambda)$$

$$E = f_\phi^2 + g_\phi^2 \quad F = f_\phi f_\lambda + g_\phi g_\lambda \quad G = f_\lambda^2 + g_\lambda^2$$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2}; \quad m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2 \cos^2 \phi}; \quad p = \frac{2(f_\phi f_\lambda + g_\phi g_\lambda)}{R^2 \cos \phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A_p'; \quad \textcolor{brown}{f}_\phi = \frac{\partial f(\phi, \lambda)}{\partial \phi}$$

$$\sin A_p' = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{(f_\lambda g_\phi - f_\phi g_\lambda)^2 + (f_\phi f_\lambda + g_\phi g_\lambda)^2}}$$

$$\text{Equivalency Condition: } f_\lambda g_\phi - f_\phi g_\lambda = \pm R^2 \cos \phi$$

$$\text{Cauchy-Riemann Equations: } f_\phi = -\frac{1}{\cos \phi} g_\lambda; \quad g_\phi = \frac{1}{\cos \phi} f_\lambda$$

$$\sin\left(\frac{\omega}{2}\right) = \frac{(a-b)}{(a+b)}$$