

CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS

March 2025

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate value to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 7 questions on 3 pages.

Marks

Q. No	Time: 3 hours	Value	Earned									
1.	The map coordinates of points R and S in a stereographic double projection and the UTM projection are as follows:											
	<table><tr><td></td><td>Stereographic double Projection</td><td>UTM projection</td></tr><tr><td>Point R:</td><td>Northing = 7,528,656.555 m, Easting = 2,497,692.180 m Latitude: 46° 45' 28.0998" N, Longitude : 66° 31' 48.7559" W Grid Convergence: -0° 01' 19.06" (T-t) Correction : 0.04"</td><td>Northing = 5,181,212.674 m, Easting = 688,610.493 m Latitude: 46° 45' 28.0998" N, Longitude: 66° 31' 48.7559" W</td></tr><tr><td>Point S:</td><td>Northing = 7,519,393.255 m, Easting = 2,498,724.831 m</td><td>Northing = 5,171,985.701 m, Easting = 689,937.055 m</td></tr></table>		Stereographic double Projection	UTM projection	Point R:	Northing = 7,528,656.555 m, Easting = 2,497,692.180 m Latitude: 46° 45' 28.0998" N, Longitude : 66° 31' 48.7559" W Grid Convergence: -0° 01' 19.06" (T-t) Correction : 0.04"	Northing = 5,181,212.674 m, Easting = 688,610.493 m Latitude: 46° 45' 28.0998" N, Longitude: 66° 31' 48.7559" W	Point S:	Northing = 7,519,393.255 m, Easting = 2,498,724.831 m	Northing = 5,171,985.701 m, Easting = 689,937.055 m		
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a) Calculate the plane bearings of line RS in the stereographic double projection and UTM projection. Explain two important reasons why the two calculated plane bearings are different.	4											
b) Calculate (independently) the geodetic bearings of line RS from the plane bearings computed for the stereographic double projection and the UTM projection (assuming the mean radius of the earth in the mapping region, $R_m = 6,382,129.599$ m). Explain one important reason why the two calculated geodetic bearings are different.	18											
2.	The map projection equations relating map projection coordinates (x, y) with the corresponding geographic coordinates (ϕ, λ) can be given as $x = R\lambda \qquad y = R \sin \phi$ where R is the mean radius of the spherical earth. Answer the following questions: a) Calculate the area distortion factor and determine (showing all your derivations and substitutions of values) if this projection is equal-area. b) Derive a mathematical expression for the maximum angular distortion at any point in the map projection and calculate the latitude where the maximum angular distortion reaches 40° on the projection.	7										
		6										
3.	According to Torge in his <i>Geodesy</i>, the Celestial Reference System (CRS) approximates an inertial system. a) What is an inertial system? Explain the need for it in Geomatics. b) Describe the three-dimensional Cartesian coordinate system and the spherical coordinate system, for CRS.	4										
		6										

4.	Explain each of the following (giving clear explanation of each of the terms involved, and do not be tempted to state, for example, that one is ... and the other is not) : a) One main difference between datum transformation and coordinate transformation. b) One main difference between reference system and reference frame. c) Compare CGVD28 with CGVD2013 by explaining how they are realized, types of height system, reference surfaces for defining them, and simple models for converting height from one system to the other (clearly showing the relationship between each system and NAD83(CSRS)).	3 3 6	
5.	Answer all the following: a) Maps that cover large areas of the globe usually have stated scale called the nominal scale. However, on such maps one simply cannot measure the distance from any point on a map to another, multiply by the nominal scale, and expect to have obtained a correct distance. Explain why. Clearly explain how you would obtain a correct distance from such maps. b) Discuss the differences and the relationship between a map projection and its map grid (describing their origins, units and axes involved).	5 6	
6.	a) Explain two fundamental roles of time in positioning applications. b) Explain the differences between the Local Astronomic coordinate system and the Horizon coordinate system regarding their origins, linear coordinate axes, fundamental planes, and the observables in each.	4 8	
7.	Using two well labelled sketches only, illustrate the Transverse Mercator (TM) and the Mercator projections in the Northern hemisphere; provide one sketch for the TM projection and the other sketch for the Mercator projection. Each sketch must show the projections of the Equator, Central Meridian (CM), not less than four parallels, not less than four meridians and not less than four Tissot indicatrices each along CM and Equator with appropriate relationship between lines of the graticule clearly illustrated	20	
		100	

Some potentially useful formulae are given as follows:

$$T_{-t} = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - 500,000$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i .

$$\text{UTM average line scale factor, } \bar{k}_{UTM} = 0.9996 \left[1 + \frac{x_u^2}{6R_m^2} \left(1 + \frac{x_u^2}{36R_m^2} \right) \right];$$

$$\text{where } x_i = x_i^{UTM} - 500,000; \quad x_u^2 = x_1^2 + x_1x_2 + x_2^2$$

Grid convergence, $\gamma_B = \Delta\lambda \sin \phi \left[1 + \frac{\Delta\lambda^2 \cos^2 \phi}{3(20265)^2} \right]$; where $\Delta\lambda = (\lambda - \lambda_0)$ (in arc-seconds)

for any given longitude λ with central longitude at λ_0 .

$$\alpha = t + \gamma + (T - t)$$

$$X_f = X_G \times K_f + X_0;$$

$$Y_f = Y_G \times K_f$$

$$Sf = \frac{R_m}{R_m + H_m}$$

$$x = (N + h) \cos \phi \cos \lambda;$$

$$y = (N + h) \cos \phi \sin \lambda$$

$$z = \left[(1 - e^2) N + h \right] \sin \phi;$$

$$N = \frac{a}{\sqrt{(1 - e^2 \sin^2 \phi)}}$$

$$\Delta r^{LG} = R_3(\eta_0 \tan \phi_0) R_2(-\xi_0) R_1(\eta_0) \times \Delta r^{LA}$$

$$\Delta r^G = R_3(\pi - \lambda_0) R_2\left(\frac{\pi}{2} - \phi_0\right) P_2 \times \Delta r^{LG}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B^G = r^G + \Delta r^G$$

$$\alpha_{ij} = A_{ij} - \eta_i \tan \phi_i - (\xi_i \sin \alpha_{ij} - \eta_i \cos \alpha_{ij}) \cot Z_{ij}$$

Distortion Formulas:

Given: $X = f(\phi, \lambda)$

$Y = g(\phi, \lambda)$

$$E = f_\phi^2 + g_\phi^2$$

$$F = f_\phi f_\lambda + g_\phi g_\lambda$$

$$G = f_\lambda^2 + g_\lambda^2$$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2}; m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2 \cos^2 \phi}; p = \frac{2(f_\phi f_\lambda + g_\phi g_\lambda)}{R^2 \cos \phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A_p'; \quad f_\phi = \frac{\partial f(\phi, \lambda)}{\partial \phi}$$

$$\sin A_p' = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{(f_\lambda g_\phi - f_\phi g_\lambda)^2 + (f_\phi f_\lambda + g_\phi g_\lambda)^2}}$$

Equivalency Condition: $f_\lambda g_\phi - f_\phi g_\lambda = \pm R^2 \cos \phi$

Cauchy-Riemann Equations: $f_\phi = -\frac{1}{\cos \phi} g_\lambda;$ $g_\phi = \frac{1}{\cos \phi} f_\lambda$

$$\sin\left(\frac{\omega}{2}\right) = \frac{(a-b)}{(a+b)}$$