

CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-3 ADVANCED SURVEYING

October 2024

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 5 questions on 5 pages.

Marks

Q. No	Time: 3 hours	Value	Earned															
1.	Your company has recently purchased a Sokkia B20 automatic level with the following manufacturer's specifications: - Standard deviation for 1 km double run leveling (according to ISO 17123-2) is ± 0.5 mm (with micrometer). - Magnification is 32$\times$; and compensator setting accuracy is $\pm 0.3''$. Answer the following regarding the level: a) Explain clearly what is meant by "1 km double run leveling" and the details behind the use of ISO 17123-2 statement in the specifications. b) Explain what $\pm 0.3''$ specified as compensator setting accuracy means and how you will use the specification. c) If the level is to be used in a survey, determine the expected standard deviation of a height difference measurement in a set-up with a sight distance of 30 m. d) If the level is to be calibrated according to ISO 17123-2, what quantities will be estimated and what statistical tests should be done on the estimated quantities (clearly providing purpose of tests, test statistics, associated statistical distributions, and possible decisions to make with all symbols defined)?	4 2 4 11																
2.	To provide control for a construction project, some leveling readings were taken over a section using precise automatic level. The Canadian First-Order specifications were followed with two Canadian First-Order benchmarks nearby used, and the collimation correction factor of the instrument determined later as +0.02 mm/m. The following were determined from the leveling field notes: <table><tr><th></th><th>Forward Run (m)</th><th>Backward Run (m)</th></tr><tr><td>Sum of BS rod readings</td><td>7.1013</td><td>12.0164</td></tr><tr><td>Sum of FS rod readings</td><td>11.5465</td><td>7.5753</td></tr><tr><td>Sum of BS distances</td><td>532.0</td><td>534.1</td></tr><tr><td>Sum of FS distances</td><td>536.8</td><td>539.1</td></tr></table> a) Calculate the elevation difference for each run (corrected for the effect of collimation error) and the mean elevation difference (in m to four decimal places) for the section. b) Determine if the leveling run satisfies the Canadian First-Order specification. c) Discuss four important aspects of the Canadian First-Order vertical control specifications that must be satisfied (including the types of errors minimized) at each leveling setup using the automatic level. d) Discuss one important disadvantage of orthometric height system and explain why surveyors prefer the system to leveled height (uncorrected differential leveling height) system (clearly discussing the differences between the two height systems).		Forward Run (m)	Backward Run (m)	Sum of BS rod readings	7.1013	12.0164	Sum of FS rod readings	11.5465	7.5753	Sum of BS distances	532.0	534.1	Sum of FS distances	536.8	539.1	5 3 8 9	
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Some potentially useful formulae are given as follows:

$$v = \frac{Z_I + Z_{II} - 360}{2} \quad \bar{z} = \frac{Z_I + (360 - Z_{II})}{2}$$

$$\frac{c}{\sin(z)} = \frac{Hz_I - (Hz_{II} - 180)}{2} \quad \frac{t}{\tan(z)} + \frac{c}{\sin(z)} = \frac{Hz_I - (Hz_{II} - 180)}{2}$$

$$\text{Corrected direction} = \text{Measured direction} - \frac{(NR - NL) \times v''}{2 \tan z}$$

$$i_v = z - z' \quad \text{or} \quad i_v = i \cos \alpha; \quad i_T = Hz - Hz' \quad \text{or} \quad i_T = \frac{i \sin \alpha}{\tan z}$$

Deformation: $\ell_2 - \ell_1 + V = Ad$;

$$d = \hat{x}_2 - \hat{x}_1$$

$$F_c = \frac{\hat{d}^T Q_{\hat{d}}^{-1} \hat{d}}{\hat{\sigma}_0^2 u_d} < F(1 - \alpha_0, u_d, df_p); \quad F_c = \frac{\hat{d}^T Q_{\hat{d}}^{-1} \hat{d}}{\hat{\sigma}_0^2 u_d} < \frac{\chi_{1-\alpha_0, df=u_d}^2}{u_d}$$

$$\alpha = \frac{\delta \Delta h}{s} \quad \text{where } \delta \Delta h = \Delta h_{12t2} - \Delta h_{12t1}$$

$$\sigma_\alpha = \frac{\sigma_{\delta \Delta h}}{s} \quad \text{where } \sigma_{\delta \Delta h} = \sqrt{\sigma_{\Delta h1}^2 + \sigma_{\Delta h2}^2}$$

EDM: $n_a = 1 + \frac{(n_g - 1)273.16p}{(273.16 + t)1013.25}$ (for p in mb and t in °C)

$$N = (n - 1) \times 10^6 \quad \delta' = (N_{REF} - N_a) d' \times 10^{-6}$$

Standard pressure: 760 mmHg or 1013.25 mb; 0°C or 273.15 K

$$\hat{C} = \frac{M - (m_1 + m_2 + m_3 + m_4 + \dots + m_n)}{n - 1}$$

Statistics:

$$|\Delta| = \sigma_\Delta \sqrt{\chi_{1-\alpha, df}^2} \quad |\Delta| \leq z_{1-\alpha/2} \sigma_\Delta \quad |\Delta| \leq t_{df, 1-\alpha/2} \sigma_\Delta \quad \hat{\sigma} \leq \sqrt{\frac{\chi_{1-\alpha, df}^2(\sigma^2)}{df}}$$

$$y = d\hat{x}^T C_{\hat{x}}^{-1} d\hat{x} \quad \chi_{\frac{\alpha}{2}, df}^2 \leq \frac{(df)s^2}{\sigma^2} \leq \chi_{1-\frac{\alpha}{2}, df}^2 \quad F_{1-\frac{\alpha}{2}, df_1, df_2} \leq \frac{s_{01}^2}{s_{02}^2} \leq F_{\frac{\alpha}{2}, df_1, df_2}$$

$$y < \chi_{u, 1-\alpha}^2 \quad a_{(1-\alpha)100\%} = a_{st} \sqrt{\chi_{1-\alpha, df}^2} \quad \text{or} \quad a_{(1-\alpha)100\%} = a_{st} \sqrt{2F_{1-\alpha, df1, df2}}$$

Error propagation: Leveling: $d\bar{h} = \frac{dh_f - dh_b}{2} \quad \sigma_{\Delta h(1km)} = \sigma_{\Delta h} \sqrt{m}$

$$\Delta h_{1km} = \Delta h_1 + \Delta h_2 + \dots + \Delta h_m$$

$$\sigma_{dp} = \frac{\sigma_p}{\sqrt{2n}} \quad \sigma_{dp} = \frac{60}{M} \quad \sigma_{\theta P} = \frac{\sigma_P}{\sqrt{n}} \quad \sigma_{dr} = \frac{\sigma_r}{\sqrt{2n}} \quad \sigma_{dr} = 2.5 \text{ div} \quad \sigma_{\theta r} = \frac{\sigma_r}{\sqrt{n}}$$

$$\sigma_L = \sigma_v \cot z, \quad \sigma_v = 0.2v'' \quad \sigma_r = 2.5d'' \quad \sigma_{\theta L} = \sigma_v \sqrt{\cot^2(Z_b) + \cot^2(Z_f)}$$

$$\sigma_i = \frac{(206265'')\sigma_{c3}}{S_1} \quad \sigma_t = \frac{(206265'')\sigma_{c1}}{S_1} \quad \sigma_{dc} = \frac{206265}{S} \sqrt{\sigma_{c3}^2 + \sigma_{c1}^2}$$

$$\sigma_c = \pm 0.5 \text{ mm/m} \times \text{HI (m)} \quad \sigma_c = \pm 0.1 \text{ mm} \quad \sigma_c = \pm 0.1 \text{ mm/m} \times \text{HI (m)}$$

$$\sigma_{\theta} = (206265'')\sigma_{c3}\sqrt{\left[\frac{S_1^2 + S_2^2 - 2S_1S_2\cos\theta}{S_1^2S_2^2}\right]}$$

$$\sigma_P = \frac{45}{206265 \times M} S; \quad \sigma_L = \left(\frac{\sigma_v}{206265}\right) S; \quad \sigma_r = \frac{\ell}{2} \left(\frac{v_r}{206265}\right)^2$$

$$\sigma_d = \frac{S}{2R} \sigma_{k_h} \quad \sigma_{ref} = \frac{S}{2R} \sigma_{k_v}$$

$$\ell = f(x) \quad C_{\hat{x}} = \sigma_0^2 (A^T P A)^{-1} \quad P = Q^{-1}$$

$$s_{\Delta x}^2 = s_{x_1}^2 + s_{x_2}^2 - 2s_{x_1x_2} \quad s_{\Delta x\Delta y} = s_{x_1y_1} + s_{x_2y_2} - s_{x_1y_2} - s_{y_1x_2} \quad s_{\Delta y}^2 = s_{y_1}^2 + s_{y_2}^2 - 2s_{y_1y_2}$$

$$\lambda_1 = \frac{1}{2}(s_{\Delta x}^2 + s_{\Delta y}^2 + R) \quad \lambda_2 = \frac{1}{2}(s_{\Delta x}^2 + s_{\Delta y}^2 - R) \quad R = \left[(s_{\Delta x}^2 - s_{\Delta y}^2)^2 + 4s_{\Delta x\Delta y}^2 \right]^{1/2}$$

$$a_s = \sqrt{\lambda_1} \quad b_s = \sqrt{\lambda_2} \quad a_{95} = k_{95}a_s \quad b_{95} = k_{95}b_s$$

$$k_{95} = \sqrt{\chi_{df,1-\alpha}^2} \quad \beta = \arctan\left(\frac{s_{\Delta x\Delta y}}{\lambda_1 - s_{\Delta x}^2}\right)$$

Map projection and Reductions:

$$\text{Meridian convergence: } \gamma = \frac{d \tan \phi (1 - e^2 \sin^2 \phi)^{1/2}}{a} \quad a = 6378137 \text{ m; } e = 0.081819191$$

$$\text{or } \gamma = L \left(1 + \frac{L^2}{3} (1 + 3\eta^2) \cos^2 \phi \right) \sin \phi$$

where $\eta^2 = e'^2 \cos^2 \phi$; $e'^2 = 0.006739496780$; $L = (\lambda - \lambda_0)$ (in radians); λ_0 is the longitude of the central meridian; and ϕ is the latitude of the given point.

$$\alpha = A - \eta \tan \phi \quad \text{where } -\eta \tan \phi \text{ is Laplace correction}$$

Horizontal Control Survey:

$$a = C(d + 0.2) \text{ cm} \quad [\text{where } d \text{ is distance in km; } C = 2 \text{ (First Order); } C = 5 \text{ (Second Order)}]$$

Vertical Control survey:

$$\pm 3mm\sqrt{L} \quad \pm 4mm\sqrt{L} \quad \pm 8mm\sqrt{L} \quad \pm 24mm\sqrt{L} \quad \pm 120mm\sqrt{L}$$

Map Accuracy Standards:

$$Accuracy_x = SE \times \sqrt{\chi_{df,1-\alpha}^2} \quad Accuracy_y = SE \times \sqrt{\chi_{df,1-\alpha}^2}$$

$$Accuracy_z = SE \times z_{1-\alpha/2} \quad CMAS = SE \times z_{1-\alpha/2}$$

$$VMAS = CI/2 \quad VMAS = SE \times z_{1-\alpha/2} \quad SE = RMSE$$

Table 1: Normal Distribution table (upper tail area):

α	0.001	0.002	0.003	0.004	0.005	0.01	0.025	0.05	0.10
z_α	3.09	2.88	2.75	2.65	2.58	2.33	1.96	1.64	1.28

Table 2: Table for Student-t distribution (α is upper tail area)

	t_{α}			
Degree of freedom	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36
6	1.49	1.94	2.45	3.14
11	1.363	1.796	2.201	2.718
12	1.356	1.782	2.179	2.681
13	1.350	1.771	2.160	2.650
14	1.345	1.761	2.145	2.624
15	1.341	1.753	2.131	2.602

Table 3: Chi-Square Distribution table (lower tail area)

α	0.025	0.05	0.10	0.90	0.95	0.975	0.99	0.995
Degrees of freedom								
1	0.001	0.004	0.016	2.705	3.841	5.024	6.635	7.879
2	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
11	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	5.009	5.892	7.041	19.811	22.362	24.736	27.688	29.819
14	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
28	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993