

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

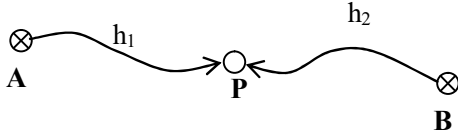
Note: This examination consists of 9 questions on 3 pages.

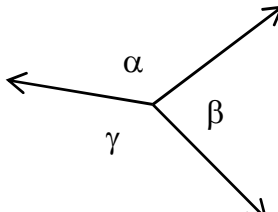
Marks

Q. No

Time: 3 hours

Value Earned

1.	Explain briefly the following: a) Standard deviation b) Circular error probability c) Root mean square error d) Correlation coefficient e) Type II error in statistical testing f) Redundancy of a linear system	15	
2.	Given the cofactor matrix Q of the horizontal coordinate (x, y) of a survey station and the unit variance $\hat{\sigma}_0^2 = 2 \text{ cm}^2$, calculate the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $Q = \begin{bmatrix} 5.32 & 6.02 \\ 6.02 & 8.38 \end{bmatrix}$	10	
3.	Given a leveling network below where A and B are known points, h_1 and h_2 are two height difference measurements with standard deviation of σ_1 and σ_2, respectively and $\sigma_1 = 2 \sigma_2$. Determine the value of σ_1 and σ_2 so that the standard deviation of the height solution at point P using least squares adjustment is equal to 2mm. 	10	
4.	Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$: $C_\ell = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$ and two functions of ℓ: $x = \ell_1 + \ell_2$ and $y = 3\ell_1$, determine $C_{xy}, C_{x\ell}, C_{y\ell}$	10	

5.	Given the angle measurements along with their standard deviations, conduct a conditional least squares adjustment. You are required to compute the following quantities: a) the estimated residuals b) the variance-covariance matrix of the estimated residuals c) the estimated observations d) the variance-covariance matrix of the estimated observations e) the estimated variance factor <table><tr><th>Angle</th><th>Measurement</th><th>Standard Deviation</th></tr><tr><td>α</td><td>134°38'56"</td><td>6.7"</td></tr><tr><td>β</td><td>83°17'35"</td><td>9.9"</td></tr><tr><td>γ</td><td>142°03'14"</td><td>4.3"</td></tr></table> 	Angle	Measurement	Standard Deviation	α	134°38'56"	6.7"	β	83°17'35"	9.9"	γ	142°03'14"	4.3"	15
Angle	Measurement	Standard Deviation												
α	134°38'56"	6.7"												
β	83°17'35"	9.9"												
γ	142°03'14"	4.3"												
6.	Conduct a parametric least squares adjustment to the same data given in Problem 5. You are required to compute the following quantities: a) the estimated parameters b) the variance-covariance matrix of the estimated parameters c) the estimated difference between α and β d) the variance of the estimated difference between α and β	10												
7.	Given the sample unit variance obtained from the adjustment of a geodetic network $\hat{\sigma}_0^2 = 0.55 \text{ cm}^2$ with a degree of freedom $\upsilon = 3$ and the a-priori standard deviation $\sigma_0 = 0.44 \text{ cm}$, conduct a statistical test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. The critical values that might be required in the testing are provided in the following table: <table><tr><th>α</th><td>0.001</td><td>0.01</td><td>0.025</td><td>0.05</td><td>0.10</td></tr><tr><th>$\chi^2_{\alpha, \upsilon=3}$</th><td>16.26</td><td>11.34</td><td>9.35</td><td>7.82</td><td>6.25</td></tr></table> where $\chi^2_{\alpha, \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha, \upsilon=3}}^{\infty} \chi^2(x)dx$.	α	0.001	0.01	0.025	0.05	0.10	$\chi^2_{\alpha, \upsilon=3}$	16.26	11.34	9.35	7.82	6.25	10
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8.	Given the following mathematical model $f(\ell, x) = 0 \quad C_\ell \quad C_x$ where f is the vector of mathematical models, x is the vector of unknown parameters and C_x is its variance matrix, ℓ is the vector of observations and C_ℓ is its variance matrix. a) Linearize the mathematical model b) Formulate the variation function	10																	
9.	The following residual vector $\hat{r}$ and estimated cofactor matrix $Q_{\hat{r}}$ were computed from a least squares adjustment using independent observations with a standard deviation of $\sigma_0 = 1.5$ mm: $\hat{r} = [4 \quad 2 \quad -3 \quad 10] \quad (\text{mm})$ $Q_{\hat{r}} = \begin{bmatrix} 15 & 1 & 3 & -2 \\ 1 & 7 & -1 & 3 \\ 3 & -1 & 4 & -1 \\ -2 & 3 & -1 & 2 \end{bmatrix} \quad (\text{mm}^2)$ Given that a global test has been rejected with a significance level of $\alpha = 0.04$, conduct further tests to identify which observation(s) may contain an outlier. The critical values that might be required in the testing are provided in the following table: <table><tr><td>α</td><td>0.001</td><td>0.002</td><td>0.003</td><td>0.004</td><td>0.005</td><td>0.01</td><td>0.05</td></tr><tr><td>K_α</td><td>3.09</td><td>2.88</td><td>2.75</td><td>2.65</td><td>2.58</td><td>2.33</td><td>1.64</td></tr></table> where K_α is determined by the equation $\alpha = \int_{K_\alpha}^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$.	α	0.001	0.002	0.003	0.004	0.005	0.01	0.05	K_α	3.09	2.88	2.75	2.65	2.58	2.33	1.64	10	
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Total Marks:		100																	