

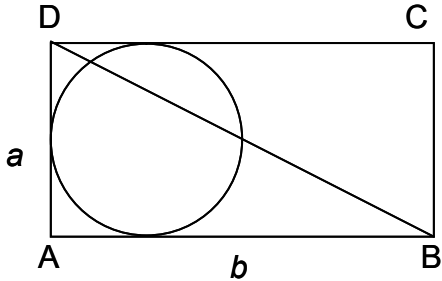
Note: This examination consists of 9 questions on 3 pages.

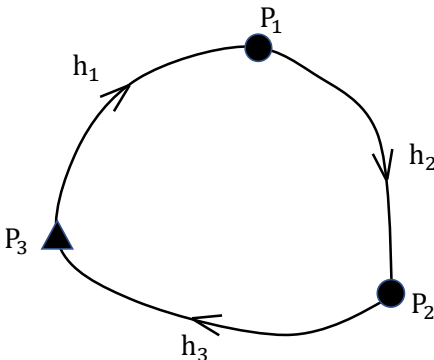
Marks

Q. No

Time: 3 hours

Value Earned

1.	Define or explain briefly the following terms: - Accuracy - Redundancy of a linear system - Correlation coefficient - External reliability - Type II error in statistical testing	10	
2	Sides a and b are measured once each as follows: $l = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix} \text{ m}$ $C_l = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \text{ cm}^2$  - Estimate the areas of triangle ABD and the circle shown inside the rectangle. - Estimate the standard deviations of the quantities computed in Part (a). - Estimate the correlation between the triangle and the circle estimates. - Discuss the nature of the correlations computed in Part (c).	15	
3.	Consider that the shape of an object is defined by the following equation: $z_i = ax_i^3 + b \sin(y_i), \quad i = 1, 2, 3.$ where z_i, x_i, y_i are observations with standard deviations $\sigma_{z_i}, \sigma_{x_i}, \sigma_{y_i}$, and a and b are parameters to be estimated. Derive the linearized form of this non-linear model for least squares adjustment including the required matrices and vectors.	10	
4.	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $C_x = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix} \text{ m}^2$	10	
5.	Prove that $\frac{\sigma}{\sqrt{n}}$ is the standard deviation of the mean value $\bar{x} = \frac{\sum_{i=1}^n \ell_i}{n}$. Each measurement ℓ_i is made with a standard deviation σ.	10	

6.	Given a leveling network (see figure below) with three height difference observations (see table below). Assume that all observations were made with the same accuracy ($\sigma = 1 \text{ mm}$). P_3 is a control point with known elevation 2.000 m. Conduct a conditional least squares adjustment on the leveling network. You are required to compute the following quantities: a) the adjusted observation residuals b) the variance-covariance matrix of the adjusted observation residuals c) the adjusted observations d) the variance-covariance matrix of the adjusted observations e) the a-posteriori variance factor  <table data-bbox="578 854 969 1026"><tr><th></th><th>Measurements</th></tr><tr><td>h_1</td><td>2.125 m</td></tr><tr><td>h_2</td><td>-1.343 m</td></tr><tr><td>h_3</td><td>-0.779 m</td></tr></table>		Measurements	h_1	2.125 m	h_2	-1.343 m	h_3	-0.779 m	15					
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7.	Conduct a parametric least squares adjustment to the same data given in Problem 6. You are required to compute the following quantities: a) the adjusted elevations b) the variance-covariance matrix of the adjusted elevations c) the adjusted observations d) the observation residuals	10													
8.	Given the sample unit variance obtained from the adjustment of a geodetic network $\hat{\sigma}_0^2 = 0.55 \text{ cm}^2$ with a degree of freedom $\nu = 3$ and the a-priori standard deviation $\sigma_0 = 0.44 \text{ cm}$, conduct a statistic test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. Provide the major test steps and explain the conclusion. The critical values that might be required in the testing are provided in the following table: <table data-bbox="331 1663 1213 1806"><tr><th>α</th><th>0.001</th><th>0.01</th><th>0.025</th><th>0.05</th><th>0.10</th></tr><tr><td>$\chi_{\alpha, \nu=3}^2$</td><td>16.26</td><td>11.34</td><td>9.35</td><td>7.82</td><td>6.25</td></tr></table>	α	0.001	0.01	0.025	0.05	0.10	$\chi_{\alpha, \nu=3}^2$	16.26	11.34	9.35	7.82	6.25	10	
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9.	A baseline of calibrated length (μ) 200.0m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean ($\bar{x}$) and sample standard deviation (s) are calculated from the measurements: $\bar{x} = 200.5\text{m} \qquad s = 0.05\text{m}$ Test at the 95% level of confidence if the measured distance is significantly different from the calibrated distance. The critical value that might be required in the testing is provided in the following table:	10																																			
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