

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate value to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

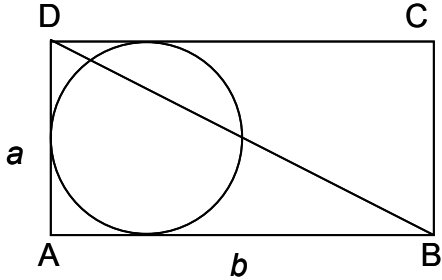
Note: This examination consists of 9 questions on 3 pages.

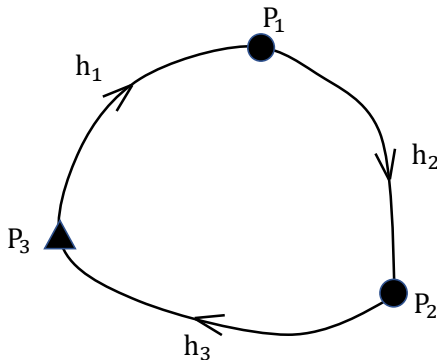
Marks

Q. No

Time: 3 hours

Value Earned

1.	Explain briefly the following terms: - Precision - Accuracy - Redundancy of a linear system - Confidence level in statistical testing - Type II error in statistical testing	10	
2.	Sides a and b are measured once each as follows: $l = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix} \text{ m}$ $C_l = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \text{ cm}^2$  - Estimate the areas of triangle ABD and the circle shown inside the rectangle. - Estimate the standard deviations of the quantities computed in Part (a). - Estimate the correlation between the triangle and the circle estimates. - Discuss the nature of the correlations computed in Part (c).	15	
3.	Consider that the shape of an object is defined by the following equation: $z_i = ax_i^3 + b \sin(y_i)$ where z_i, x_i, y_i are observations with standard deviations $\sigma_{z_i}, \sigma_{x_i}, \sigma_{y_i}$, and a and b are parameters to be estimated. Assume $i = 1, 2, 3$. Write the linearized form of this model and derive the required matrices and vectors.	10	

4.	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $C_x = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix} \text{ m}^2$	10									
5.	Prove that $\frac{\sigma}{\sqrt{n}}$ is the standard deviation of the mean value $\bar{x} = \frac{\sum_{i=1}^n \ell_i}{n}$, each measurement ℓ_i is made with a standard deviation σ.	10									
6.	Given a leveling network (see figure below) with three height difference observations (see table below). Assume that all observations were made with the same accuracy ($\sigma = 1 \text{ mm}$). P_3 is a control point with known elevation 2.000 m. Conduct a conditional least squares adjustment on the leveling network. You are required to compute the following quantities: a) the adjusted observation residuals b) the variance-covariance matrix of the adjusted observation residuals c) the adjusted observations d) the variance-covariance matrix of the adjusted observations e) the a-posteriori variance factor  <table><thead><tr><th></th><th>Measurements</th></tr></thead><tbody><tr><td>h_1</td><td>2.125 m</td></tr><tr><td>h_2</td><td>-1.343 m</td></tr><tr><td>h_3</td><td>-0.779 m</td></tr></tbody></table>		Measurements	h_1	2.125 m	h_2	-1.343 m	h_3	-0.779 m	15	
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7.	Conduct a parametric least squares adjustment to the same data given in Problem 6. You are required to compute the following quantities: a) the adjusted elevations b) the variance-covariance matrix of the adjusted elevations c) the adjusted observations d) the observation residuals	10																																				
8.	Given the sample unit variance obtained from the adjustment of a geodetic network $\hat{\sigma}_0^2 = 0.55\text{ cm}^2$ with a degree of freedom $\nu = 3$ and the a-priori standard deviation $\sigma_0 = 0.44\text{ cm}$, conduct a statistic test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. Provide the major test steps and explain the conclusion. The critical values that might be required in the testing are provided in the following table: <table><tr><td>α</td><td>0.001</td><td>0.01</td><td>0.025</td><td>0.05</td><td>0.10</td></tr><tr><td>$\chi^2_{\alpha, \nu=3}$</td><td>16.26</td><td>11.34</td><td>9.35</td><td>7.82</td><td>6.25</td></tr></table>	α	0.001	0.01	0.025	0.05	0.10	$\chi^2_{\alpha, \nu=3}$	16.26	11.34	9.35	7.82	6.25	10																								
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9.	A baseline of calibrated length (μ) 200.0m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean ($\bar{x}$) and sample standard deviation (s) are calculated from the measurements: $\bar{x} = 200.5\text{m}$ $s = 0.05\text{m}$ Test at the 95% level of confidence if the measured distance is significantly different from the calibrated distance. The critical value that might be required in the testing is provided in the following table: <table><tr><td></td><td colspan="4">t_{α}</td></tr><tr><td>Degree of freedom</td><td>$t_{0.90}$</td><td>$t_{0.95}$</td><td>$t_{0.975}$</td><td>$t_{0.99}$</td></tr><tr><td>1</td><td>3.08</td><td>6.31</td><td>12.7</td><td>31.8</td></tr><tr><td>2</td><td>1.89</td><td>2.92</td><td>4.30</td><td>6.96</td></tr><tr><td>3</td><td>1.64</td><td>2.35</td><td>3.18</td><td>4.54</td></tr><tr><td>4</td><td>1.53</td><td>2.13</td><td>2.78</td><td>3.75</td></tr><tr><td>5</td><td>1.48</td><td>2.01</td><td>2.57</td><td>3.36</td></tr></table>		t_{α}				Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	1	3.08	6.31	12.7	31.8	2	1.89	2.92	4.30	6.96	3	1.64	2.35	3.18	4.54	4	1.53	2.13	2.78	3.75	5	1.48	2.01	2.57	3.36	10	
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