

CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-1 MATHEMATICS

October 2025

Although programmable calculators may be used, when you answer these questions, show your work and do not use fancy calculator functions (calculus or linear algebra, such as derivatives, integrals, determinants, inverses, eigenvalues, etc.)

Note: This examination consists of 10 questions on 3 pages.

Marks

Q. No

Time: 3 hours

Value Earned

1.	(a) Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{pmatrix} 5 & 2 \\ 3 & 4 \end{pmatrix}$ (b) Now find P and D such that $A = PDP^{-1}$	10	
2.	Find the Taylor polynomial of degree 2 for $f(x) = \ln(x)$ centered at $a = 1$.	10	
3.	A land surveyor is measuring property boundaries using a theodolite, but atmospheric refraction causes cumulative measurement errors that compound over distance. On the first 100-meter segment, the cumulative error is $a_0 = 2.4$ cm. On the second 100-meter segment, the additional cumulative error is $a_1 = 1.8$ cm. The error continues to accumulate with each subsequent 100-meter segment contributing a_{n-1} cm of additional error in the n-th segment such that $\sum_{n=0}^{\infty} a_n$ forms a geometric series. The surveyor's equipment has a tolerance limit of 8.0 cm total cumulative error before requiring recalibration. Find the maximum total cumulative error following this pattern. Will the surveyor need to recalibrate before completing the survey? If the series diverges, you can indicate that the total error is ∞. Reminder: the formula for the limit of a geometric series is $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \text{ for } r < 1$	10	
4.	(a) Given the matrix $A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & a & 5 \\ 1 & 4 & -2 \end{pmatrix}$ Find the determinant (8 points). (b) Find the trace. The trace of a matrix is the sum of its diagonal elements. (2 points)	10	

5.	Let's find i^i together. Locate i in a 2-dimensional coordinate system where 1 is at (1, 0) and i is at (0, 1). Then rewrite the base of i^i (not the exponent) in polar form, where $a + bi$ is rectangular form and $re^{i\theta}$ is polar form (r stands for radius, θ stands for the angle). Now simplify $(re^{i\theta})^i$ using precalculus rules for exponents and find i^i—it's a real number!	10	
6.	Using basic rules of differentiation, the derivative of $f(x) = \sqrt{2x+5}$ is $f'(x) = \frac{1}{\sqrt{2x+5}}$. I want you to find this derivative using the definition of a derivative, $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ Hint: use multiplying by the conjugate, something similar to $\frac{\sqrt{a} - \sqrt{b}}{c} = \frac{(\sqrt{a} - \sqrt{b}) \cdot (\sqrt{a} + \sqrt{b})}{c \cdot (\sqrt{a} + \sqrt{b})} = \frac{a - b}{c \cdot (\sqrt{a} + \sqrt{b})}$	10	
7.	(a) Find $\int_0^{\frac{\pi}{2}} \sin(2x) dx$ by substituting $u = 2x$. (b) Find $\int_0^{\frac{\pi}{2}} \sin(2x) dx$ again, this time by using the double angle formula $\sin(2\theta) = 2 \sin \theta \cos \theta$ and substituting $u = \sin x$.	10	
8.	(a) Find the angle (express your answer in degrees) between the vectors $\vec{a} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \vec{b} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ (b) Find a unit vector that is orthogonal to both $\vec{a}$ and $\vec{b}$. No rounding.	10	
9.	A geomatics surveyor is conducting a topographic survey and needs to determine the elevations of three reference points: A, B, and C. Using differential leveling techniques, the surveyor establishes the following relationships between the elevations: From benchmark measurements: 2 times the elevation of point A plus 3 times the elevation of point B plus the elevation of point C equals 950.0 meters. From traverse calculations: The elevation of point A plus 4 times the elevation of point B plus 2 times the elevation of point C equals 1000.0 meters. From triangulation measurements: 3 times the elevation of point A plus the elevation of point B plus 2 times the elevation of point C equals 950.0 meters. Find the elevation of each reference point.	10	

10.	(a) The surveyor's EDM (Electronic Distance Meter) measures distances using phase shift analysis. The phase relationship is given by: $\phi(d) = \arctan\left(\frac{2d}{15}\right)$ where d is distance in meters and ϕ is phase shift in radians. Find the inverse function $d(\phi)$ to convert phase measurements back to distances. (b) The dam's spillway follows a parabolic curve described by: $y = 0.02x^2 - 0.8x + 50$ where coordinates are in meters. Find x and y coordinates of the vertex of this parabola (the lowest point of the spillway).	10	
	Total Marks:	100	