

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 10 questions on 2 pages.

Marks

Some information you may find helpful:

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \sec x = \tan x \sec x$$

$$\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\sin\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

<u>Q. No</u>	<u>Time: 3 hours</u>	<u>Value</u>	<u>Earned</u>
1.	(a) Find the following limit. $\lim_{x \rightarrow -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4}$ (b) Find the following limit. $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$	10	
2.	Consider the following quadratic equation: $(k + 1)x^2 + (2k - 4)x + k = 4$ There is only one value for k for which this equation has exactly one solution. Find this value of k and then use it to solve the equation for x .	10	
3.	Solve the following initial value problem for $y(x)$: $y' = xy, y(\sqrt{2}) = 2e$	10	

4.	Using Taylor polynomials, identify A, B, C, D in the approximation $\tan x \approx A + B \left(x - \frac{\pi}{4}\right) + C \left(x - \frac{\pi}{4}\right)^2 + D \left(x - \frac{\pi}{4}\right)^3$	10	
5.	(a) Use the substitution $u = 3x^2 - 2$ to evaluate the following definite integral: $\int_1^3 x \sqrt{3x^2 - 2} \, dx$ (b) The conjugate of $a - b$ is $a + b$. Multiplying by the conjugate has an interesting effect: $(a - b)(a + b) = a^2 - b^2$. Expand the fraction in the following integral by the conjugate in order to solve it. $\int_0^{\frac{\pi}{4}} \frac{1}{1 - \sin x} \, dx$	10	
6.	Consider the matrix $M = \begin{bmatrix} x & x + y \\ x - y & x \end{bmatrix}$ Find the characteristic polynomial $p(\lambda)$ of M (setting the characteristic polynomial to zero gives you the eigenvalues of M) in terms of x and y . Simplify as much as possible.	10	
7.	If $g(\vartheta) = \vartheta \sin \vartheta$, find $g''(\pi/6)$ without rounding.	10	
8.	Solve the following right spherical triangle with a right angle at A for the sides b, c and the angle C . $a = 109^\circ 17', B = 37^\circ 15'$	10	
9.	The solution for this question may look a bit awkward, but I want you to find it precisely (without rounding). Find the equation of the tangent line for the following function at $a = \frac{\pi}{4}$, $f(x) = \frac{9}{5} \cos \left(3x + \frac{7\pi}{12} \right)$	10	
10.	Find the determinant of the following matrix in terms of k : $A = \begin{bmatrix} 3 & -1 & 0 \\ k & 2 & 2 \\ -4 & 1 & -2 \end{bmatrix}$	10	
Total Marks:		100	