CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS <u>October 2021</u>

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 5 questions on 3 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	Scientists prefer to work with geodetic coordinates (latitude, longitude) as reference system compared to map projection coordinates (northing, easting). Explain five important advantages and five important disadvantages of using geodetic coordinates as nation-wide grid reference system compared with map projection coordinates.	15	
2.	Answer all of the following: a) Using a well labeled sketch only, illustrate the Mercator projection in the Northern hemisphere. The sketch must show the representation (in dotted lines) of loxodrome with bearing 90˚, and the projection (in bold lines) of the Equator, Central Meridian, parallels and meridians with the appropriate relationship between the lines of the graticule clearly illustrated. b) Discuss briefly the concept of Tissot indicatrix and describe the characteristics of Tissot indicatrices (with consideration for their sizes and shapes) as they will appear when drawn along the Equator and along the Central Meridian of each of the following map projections: Lambert Conformal Conic (one standard parallel), Mercator and UTM (identifying also the regions with perfect indicatrix in each case). c) Explain how the easting and northing coordinates, meridian convergence and scale factor variations in Transverse Mercator projections mathematically relate to those in the Universal Transverse Mercator projections. d) Give two important reasons why the conformality property of UTM projection is attractive to surveyors.	 8 15 5 4	
3.	Answer all of the following: a) Explain two important reasons why GPS coordinate values, which are usually presented in (X, Y, Z) geocentric coordinate system, are inconvenient for use by surveyors. b) You are required to transform three-dimensional baseline vectors determined in GNSS survey to map projection coordinates. Explain (without providing specific formulae) the steps for the transformation, indicating types of surfaces, types of coordinates and types of transformation functions involved in each of the steps. c) Name one inertial reference system (or an approximation), explain why it is an inertial system, and discuss how its curvilinear coordinates are measured (including their units and zero points of the coordinates). d) Discuss three classes of time scales that are each related to some natural observable phenomenon (stating the phenomena they are related to and how).	 4 6 5 3	

4.	The point scale factor (k) and the meridian convergence (C) at any given point (ϕ, λ) on a UTM projection can be approximated using the following formulas: $k = k_0\left[1 + \frac{(L\cos\phi)^2}{2}\right]$; $\tan(C) = -(\sin\phi)\times\tan(\lambda - \lambda_{CM})$ where $L = (\lambda - \lambda_{CM})$ expressed in radians; λ_{CM} is the longitude of the central meridian; k_0 is the scale factor at the central meridian; and ϕ and λ are the latitude and longitude values of the given point. Answer the following:		
	a) At what distance (in degrees, minutes, seconds) away from the central meridian, along the equator, is the UTM scale distortion equal to zero?	4	
	b) If a scaling accuracy ratio of 1:10,000 is to be maintained in the given zone and a modified Transverse Mercator (MTM) projection (similar to UTM) is to be used, determine minimum and maximum scale factors and the maximum width (in degrees, minutes, seconds) of the zone, at the equator.	5	
	c) Given the central meridian of a UTM zone 10 as 123° W; the geodetic coordinates of point B as Latitude = 50°00′0.000″ N, Longitude = 124°00′10.000″ W; and the corresponding UTM coordinates of point B as Northing = 5539112.50 m, Easting = 428134.53 m; answer the following:		
	(i) Calculate (on the UTM plane) the meridian convergence (to the nearest arc seconds) and the point scale factor (to six decimal places) for point B. Would this convergence angle change for the MTM zone if the MTM projection zone and the UTM zone have the same central meridian? Clearly explain your answers.	6	
	(ii) If the modified Transverse Mercator (MTM) projection zone designed in Question (b) and the UTM zone in Question (c) have the same central meridian, what are the MTM coordinates of point B (assuming the MTM False Easting = 4,500,000.00 m, False Northing = 0.00 m)?	10	
5.	The map projection equations relating map projection coordinates (x, y) with the corresponding geographic coordinates (ϕ, λ) can be given as $x = R\lambda$ $\qquad$ $y = R\sin\phi$ where R is the mean radius of the spherical earth. Determine (showing all of your mathematical derivations and substitutions of values) if the projection is equivalent using two different approaches (the conclusions of the two approaches must be identical).	10	
		100	

Some potentially useful formulae are given as follows:

$$\textit{Arc-to-chord correction} = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - x_0$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i.

UTM average line scale factor, $\bar{k}_{UTM} = k_0\left[1 + \frac{x_u^2}{6R_m^2}\left(1 + \frac{x_u^2}{36R_m^2}\right)\right]$;

where $x_i = x_i^{UTM} - x_0$; $\quad x_u^2 = x_1^2 + x_1x_2 + x_2^2$

Transformation Formulas:

$$X_{(target)} = k_{0(target)} X_G + X_{0(target)}$$
$$Y_{(target)} = k_{0(target)} Y_G$$
$$X_G = \frac{\left[X_{(original)} - X_{0(original)}\right]}{k_{0(original)}}$$
$$Y_G = \frac{Y_{(original)}}{k_{0(original)}}$$

Typical values for k_0 = 0.9996 and X_0 = 500,000 m

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$$\mathbf{r}(t) = \mathbf{r}_0 + \dot{\mathbf{r}}\,(t - t_0)$$

where $\mathbf{r}_0$ and $\dot{\mathbf{r}}$ are the position and velocity respectively at $\mathbf{t}_0$.

Distortion Formulas:

Given: $X = f(\phi, \lambda)$ $\quad$ $Y = g(\phi, \lambda)$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2}; \; m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2 \cos^2 \phi}; \; p = \frac{2\left(f_\phi f_\lambda + g_\phi g_\lambda\right)}{R^2 \cos\phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A_p' \;;$$

$$\sin A_p' = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{\left(f_\lambda g_\phi - f_\phi g_\lambda\right)^2 + \left(f_\phi f_\lambda + g_\phi g_\lambda\right)^2}}$$

Equivalency Condition: $f_\lambda g_\phi - f_\phi g_\lambda = \pm R^2 \cos\phi$

Cauchy-Riemann Equations: $f_\phi = -\dfrac{1}{\cos\phi} g_\lambda \;;$ $\qquad g_\phi = \dfrac{1}{\cos\phi} f_\lambda$