CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-3 ADVANCED SURVEYING **October 2021**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 4 questions on 4 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	Using modern total stations [angular accuracy of ± 1″ and a distance accuracy of ± 1 mm ± 1 ppm according to ISO Standards] has the potential for competing with precise differential levelling. Assuming the total station is used in a leap-frog trigonometric levelling procedure with imposed balanced maximum sight length of 150 m, determine numerically (stating any assumptions made) whether the levelling result will satisfy the First Order specification. [Let the average zenith angle at each station to back sight target be 91.5° and to foresight target as 88.5°, and assume that the standard deviation of height difference measurement between back sight and foresight targets at each setup is zero.]	15	
2.	Answer the following: a) A closed-loop horizontal traverse of 4 points is to be run in a fairly flat and homogeneous terrain using Leica 802 total station with specified standard deviation of 2″ according to ISO 17123-3 Standard. Assume each traverse leg is approximately 100 m long; the included angle at each traverse point is approximately 90°; the targets and total station are to be centered on tripods using tribrachs with optical plummets; heights of targets and instrument are to be set at 1.6 m and the included angle at each traverse point is to be measured in one set. With consideration for leveling, centering, pointing and reading errors at each setup point, determine if the traverse will satisfy the allowable angular misclosure of 15″ at 95% confidence level. b) Discuss the differences between closed-loop and closed connecting horizontal traverse network surveys and explain specifically (describing the error sources) why ROM (ratio of misclosure) of one of the traverse surveys (identifying which one) will represent precision rather than relative accuracy of the survey. c) In a 3D loop traverse survey, the covariance matrix ($C_{\hat{x}}$) of the (x, y, z) coordinates of the last traverse point and the vector of coordinate differences ($d\hat{x}$) between the published and computed coordinates of the last traverse point are as follows: $$C_{\hat{x}} = \begin{bmatrix} 1.01569E-5 & 4.53695E-6 & -3.82173E-6 \\ 4.53695E-6 & 1.25310E-5 & -1.10527E-6 \\ -3.82173E-6 & -1.10527E-6 & 3.25053E-5 \end{bmatrix}$$ $$d\hat{x} = \begin{bmatrix} 0.0042 \\ 0.0038 \\ -0.0062 \end{bmatrix}$$ Test statistically if the computed 3D coordinates of the last traverse point is compatible with the published coordinates at 99% confidence level.	14 7 5	

3.	A Leica/Wild Di1600 electro-optical distance measuring instrument (EODMI) is to be checked for its accuracy using a nearby government calibration baseline (of Heerbrugg design) of seven pillars with known mark-to-mark distances. The manufacturer's claimed accuracy of the instrument is 3 mm ± 2 ppm. Answer the following with regard to ISO 17123-4 calibration procedure: a) Explain all of the necessary setting out properties of the baseline pillars according to Heerbrugg design, seven important Quality Assurance (QA) /Quality Control (QC) measures (including their purposes), and the field procedure (including the number and types of measurements made in the field) that you would recommend following in order to determine whether the EDM is capable of behaving as the manufacturer claimed. b) Explain what reductions must be done on the measurements as "pre-processing" and why. c) Explain four quantities that will be determined from the processing of the measurements and fully discuss the statistical tests that will be performed on the quantities in order to determine whether the EDM is capable of behaving as the manufacturer claimed. d) Assuming the government-provided distances for the baseline are considered errorless and the reduced calibration measurements are to be adjusted by parametric least squares method, explain the parametric model equations (in vector form) to be used, defining the symbols representing the parameters and measurements and how the calibration values will be estimated from the adjusted parameters.	16 2 6 5	
4.	a) Explain the important differences in the First-order Design (FOD) objectives of deformation monitoring network and geodetic positioning network. b) In two-epoch method of deformation analysis what statistical test(s) must be performed (clearly stating the statistics, quantities tested and purpose) on each epoch measurements and in the deformation analysis? c) You are to perform SOD for horizontal positioning using the process of simulation. The relative positioning tolerance (limit on relative error ellipses at 95% confidence level) is to be 0.010 m and the potential observables will be angles and distances. Explain step-by-step and logically your whole plan on how to perform the simulation. Your plan must be complete enough for a computer programmer to use in developing a software application for the network design (do not be tempted to say, "Just use Pre-analysis or Design software"). You must provide the following for full marks: interpretation of the given tolerance, how to select potential geometry; input to the design and their underlying equations; design equations; general computational steps and appropriate equations and matrix expressions used at each step of the process (refer to the attached formula sheets for relevant equations and matrix expressions). Marks will be awarded depending on your demonstration of full understanding of the process involved and how much relevant details are logically provided.	4 6 20	
		100	

Some potentially useful formulae are given as follows:

$$v = \frac{Z_I + Z_{II} - 360}{2} \qquad \bar{z} = \frac{Z_I + (360 - Z_{II})}{2}$$

$$\frac{c}{\sin(z)} = \frac{Hz_I - (Hz_{II} - 180)}{2} \qquad \frac{t}{\tan(z)} + \frac{c}{\sin(z)} = \frac{Hz_I - (Hz_{II} - 180)}{2}$$

$$\text{Corrected direction} = \text{Measured direction} - \frac{(NR - NL) \times v''}{2 \tan z}$$

$$i_v = z - z' \quad \text{or} \quad i_v = i \cos\alpha; \qquad i_T = Hz - Hz' \quad \text{or} \quad i_T = \frac{i \sin\alpha}{\tan z}$$

Deformation: $\ell_2 - \ell_1 + V = Ad$; $\qquad d = \hat{x}_2 - \hat{x}_1$

$$F_c = \frac{\hat{d}^T Q_{\hat{d}}^{-1} \hat{d}}{\hat{\sigma}_0^2 u_d} < F(\alpha_0, u_d, df_p); \qquad F_c = \frac{\hat{d}^T Q_{\hat{d}}^{-1} \hat{d}}{\hat{\sigma}_0^2 u_d} < \frac{\chi^2_{\alpha_0, df=u_d}}{u_d}$$

$\alpha = \frac{\delta\Delta h}{s}$ where $\delta\Delta h = \Delta h_{12t2} - \Delta h_{12\,t1}$.

$\sigma_\alpha = \frac{\sigma_{\delta\Delta h}}{s}$ where $\sigma_{\delta\Delta h} = \sqrt{\sigma^2_{\Delta h1} + \sigma^2_{\Delta h2}}$

EDM:

$n_a = 1 + \frac{(n_g - 1)273.16p}{(273.16 + t)1013.25}$ (for p in mb and t in °C)

$N = (n-1) \times 10^6 \qquad \delta' = (N_{REF} - N_a)d' \times 10^{-6}$

Standard pressure: 760 mmHg or 1013.25 mb; 0°C or 273.15 K

$$\hat{C} = \frac{M - (m_1 + m_2 + m_3 + m_4 + ... + m_n)}{n-1}$$

Levelling: $\pm 3mm\sqrt{L} \qquad \pm 4mm\sqrt{L} \qquad \pm 8mm\sqrt{L} \qquad \pm 24mm\sqrt{L} \qquad \pm 120mm\sqrt{L}$

Statistics:

$$|\Delta| = \sigma_\Delta \sqrt{\chi^2_{df,\alpha}} \qquad |\Delta| \le z_{\alpha/2}\sigma_\Delta \qquad |\Delta| \le t_{df,\alpha/2}\sigma_\Delta \qquad \hat{\sigma} \le \sqrt{\frac{\chi^2_{\alpha,df}(\sigma)}{df}}$$

$$y = d\hat{x}^T C_{\hat{x}}^{-1} d\hat{x} \qquad y < \chi^2_{u,1-\alpha}$$

$$\sigma_{dp} = \frac{\sigma_p}{\sqrt{2n}} \qquad \sigma_{dp} = \frac{60}{M} \qquad \sigma_{\theta P} = \frac{\sigma_P}{\sqrt{n}} \qquad \sigma_{dr} = \frac{\sigma_r}{\sqrt{2n}} \qquad \sigma_{dr} = 2.5 \text{ div} \qquad \sigma_{\theta r} = \frac{\sigma_r}{\sqrt{n}}$$

$$\sigma_L = \sigma_v \cot z, \qquad \sigma_v = 0.2v'' \qquad \sigma_r = 2.5d'' \qquad \sigma_{\theta L} = \sigma_v \sqrt{\cot^2(Z_b) + \cot^2(Z_f)}$$

$$\sigma_i = \frac{(206265'')\sigma_{c3}}{S_1} \qquad \sigma_t = \frac{(206265'')\sigma_{c1}}{S_1} \qquad \sigma_{dc} = \frac{206265}{S}\sqrt{\sigma^2_{c3} + \sigma^2_1}$$

$$\sigma_c = \pm 0.5mm/\text{m} \times \text{HI (m)} \qquad \sigma_c = \pm 0.1 \text{ mm} \qquad \sigma_c = \pm 0.1 \text{ mm/m} \times \text{HI (m)}$$

$$\sigma_{\theta i} = (206265'')\sigma_{c3}\sqrt{\left[\frac{S_1^2 + S_2^2 - 2S_1S_2\cos\theta}{S_1^2 S_2^2}\right]}$$

$$\sigma_{\theta i+t} = (206265'')\sqrt{\frac{\sigma^2_{c1}}{S_1^2} + \frac{\sigma^2_{c2}}{S_2^2} + \frac{\sigma^2_{c3}}{S_1^2 S_2^2}\left[S_1^2 + S_2^2 - 2S_1S_2\cos\theta\right]}$$

$$\sigma_P = \frac{45}{206265 \times M} S; \qquad \sigma_L = \left(\frac{\sigma_v}{206265}\right) S; \qquad \sigma_r = \frac{\ell}{2}\left(\frac{v_r}{206265}\right)^2$$

$$\sigma_d = \frac{S}{2R}\sigma_{k_h} \qquad \sigma_{ref} = \frac{S}{2R}\sigma_{k_V}$$

$$\ell = f(x) \qquad C_{\hat{x}} = \sigma_0^2 \left(A^T P A\right)^{-1} \qquad \mathrm{P} = Q^{-1}$$

$$s_{\Delta x}^2 = s_{x_1}^2 + s_{x_2}^2 - 2s_{x_1 x_2} \qquad s_{\Delta x \Delta y} = s_{x_1 y_1} + s_{x_2 y_2} - s_{x_1 y_2} - s_{y_1 x_2} \qquad s_{\Delta y}^2 = s_{y_1}^2 + s_{y_2}^2 - 2s_{y_1 y_2}$$

$$\lambda_1 = \frac{1}{2}\left(s_{\Delta x}^2 + s_{\Delta y}^2 + R\right) \qquad \lambda_2 = \frac{1}{2}\left(s_{\Delta x}^2 + s_{\Delta y}^2 - R\right) \qquad R = \left[\left(s_{\Delta x}^2 - s_{\Delta y}^2\right)^2 + 4s_{\Delta x \Delta y}^2\right]^{1/2}$$

$$a_s = \sqrt{\lambda_1} \qquad b_s = \sqrt{\lambda_2} \qquad a_{95} = k_{95} a_s \qquad b_{95} = k_{95} b_s$$

$$k_{95} = \sqrt{\chi^2_{2,1-0.05}} \qquad \beta = \arctan\left(\frac{s_{\Delta x \Delta y}}{\lambda_1 - s_{\Delta x}^2}\right)$$

Table 1: Normal Distribution table (upper tail area):

α	0.001	0.002	0.003	0.004	0.005	0.01	0.025	0.05	0.10
z_α	3.09	2.88	2.75	2.65	2.58	2.33	1.96	1.64	1.28

Table 2: Chi-Square Distribution table (lower tail area)

α	0.025	0.05	0.10	0.90	0.95	0.975	0.99	0.995
Degrees of freedom								
1	0.001	0.004	0.016	2.705	3.841	5.024	6.635	7.879
2	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
11	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	5.009	5.892	7.041	19.811	22.362	24.736	27.688	29.819
14	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801

Table 3: Table for Student-t distribution (α is upper tail area)

	t_α			
Degree of freedom	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36
6	1.49	1.94	2.45	3.14
11	1.363	1.796	2.201	2.718
12	1.356	1.782	2.179	2.681
13	1.350	1.771	2.160	2.650
14	1.345	1.761	2.145	2.624
15	1.341	1.753	2.131	2.602