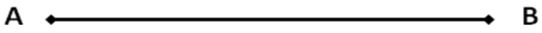
CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C2 - LEAST SQUARES ESTIMATION & DATA ANALYSIS October 2021

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 10 questions on 4 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	Define and explain the following: a) Difference between precision and accuracy b) Difference between root mean square error and standard deviation c) Difference between covariance and correlation coefficient d) Internal and external reliability e) Type I and type II errors in statistical testing	15	
2.	The distance between Point A and Point B has been independently measured 5 times with the same precision using a distance measuring device and the standard deviation of the obtained mean distance is 1.58 cm. Determine the precision of the distance measurement. 	5	
3.	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $C_x = \begin{bmatrix} 0.0484 & 0.0246 \\ 0.0246 & 0.0196 \end{bmatrix} \ m^2$	10	

4.

Given the following mathematical model

$$f(l, x) = 0 \qquad C_l \quad C_x$$

where f is the vector of mathematical models, x is the vector of unknown parameters and C_x is its variance matrix, l is the vector of observations and C_l is its variance matrix.

a) Linearize the mathematical model
b) Formulate the variation function
c) Derive the least squares normal equation

15

5.

Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$:

$$C_\ell = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

and the function $x = \ell_1 + \ell_2$, determine C_x.

5

6.

An angle has been measured independently 5 times with the same precision and the observed values are given in the following table. Test at the 95% level of confidence if the sample mean is significantly different from the true angle value 45°00′00″.

α_1	α_2	α_3	α_4	α_5
45°00′05″	45°00′10″	44°59′58″	45°00′07″	44°59′54″

The critical value that might be required in the testing is provided in the following table:

	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

10

7.

A distance has been independently measured 4 times and its sample unit variance obtained from the adjustment $\hat{\sigma}_0^2$ is equal to 1.44 cm. If the a-priori standard deviation σ_0 is 1.0 cm, conduct a statistic test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. The critical values that might be required in the testing are provided in the following table:

α	0.001	0.01	0.025	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=3}$	16.26	11.34	9.35	7.82	6.25

where $\chi^2_{\alpha,\ \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=3}}^{\infty} \chi^2(x)dx$ and υ is the degree of freedom.

10

8.

Given a geodetic network with 100 observations and 50 unknown points, use mathematical equations to explain which method of adjustment (parametric or conditional) you will recommend for this problem.

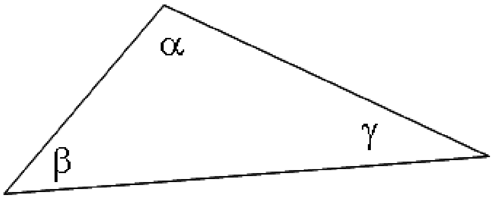
5

9.

Given the angle measurements of a triangle along with their standard deviations, conduct a conditional least squares adjustment. You are required to compute the following quantities:

a) the estimated residuals
b) the variance-covariance matrix of the estimated residuals
c) the estimated observations
d) the variance-covariance matrix of the estimated observations
e) the estimated variance factor

Angle	**Measurement**	**Standard Deviation**
α	104°38’56”	6.7”
β	43°17’35”	9.9”
γ	32°03’14”	4.3”



15

10.	Conduct a parametric least squares adjustment to the same data given in Problem 9. You are required to compute the following quantities: a) the estimated parameters b) the variance-covariance matrix of the estimated parameters c) the estimated difference between α and β d) the variance of the estimated difference between α and β	10	
	Total Marks:	100	