CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-1 MATHEMATICS

October 2021

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 10 questions on 2 pages.

Time: 3 hours

Q. No		Marks Value	Earned
1.	a) What is the limit of $x^{-1}\sin x$ as $x \to 0$? Show all the steps in deriving the limit. b) What is the limit of $\frac{\ln x}{1-x}$ as $x \to 1$? Show all the steps in deriving the limit.	10	
2.	a) Find the Taylor polynomials of orders 0, 1, 2, 3 generated by $f(x) = \sin x$ at $a = \frac{\pi}{4}$. b) Find the Mclaurin series for $f(x) = e^{\frac{x}{2}}$.	10	
3.	Solve the following spherical triangle: $a = 122°18', b = 88°21', C = 100°16'$. Note that a is the length of a side provided as an angle, while C is an angle of the spherical triangle. Solving a triangle means finding all unknown sides and angles of the spherical triangle.	10	
4.	a) Find the interior angles in degrees for the triangle spanned by the three points $P = (1, 4, -2), Q = (-1, 1, 2), R = (-1, 3, 1)$. You may want to use the dot product. b) Find the plane equation for the plane containing this triangle. You may want to use the cross product.	10	
5.	Given two arbitrary points P and Q on the surface of the Earth, what is the spherical distance between them given their respective geocentric latitude ϕ and longitude λ?	10	
6.	a) Given the differential equation: $\frac{du}{dx} + u = 3$, what is the general solution? b) Given another differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$, what is the general solution? Hint: try writing $y(x) = e^{rx}$.	10	
7.	a) Given the complex polynomial equation $z^n - 1 = 0$ for $n = 2, 3, 4$, what are the corresponding roots? b) Find the determinant of the following matrix: $A = \begin{bmatrix} 1-4i & 3-i \\ -3i & 3+4i \end{bmatrix}$	10	

8.	a) What is the gradient of the function $f(x, y, z) = \sin x + 2\cos y + 3\tan z$? Hint: the gradient is defined to be $\nabla f(p) = \begin{pmatrix} \frac{\partial f}{\partial x_1}(p) \\ \vdots \\ \frac{\partial f}{\partial x_n}(p) \end{pmatrix}$. b) What is the Laplacian of the same function? Hint: the Laplacian is defined to be $\Delta f = \nabla \cdot \nabla f = \sum_{i=1}^{n} \frac{\partial^2 f}{\partial x_i^2}$.	10	
9.	A telephone line hangs between two poles 14 metres apart in the shape of a catenary $y = 20\cosh\left(\frac{x}{20}\right) - 15$, where x and y are measured in metres. Find the length of telephone wire needed between the two poles. Note that the formula for arc length is $L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$, that $\cosh x$ and $\sinh x$ are each other's derivative, and that $\cosh^2 x - \sinh^2 x = 1$.	10	
10.	a) Find the equation of the tangent line at $(\pi/3, 2)$ for $y = \sec x$. b) What is the curvature of $f(t) = \sin t$ at $t = \frac{\pi}{4}$? Note that the formula for curvature is $\kappa = \frac{\lvert f''(t)\rvert}{(1 + (f'(t))^2)^{\frac{3}{2}}}$.	10	
	Total Marks:	100	