CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS **March 2024**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 5 questions on 3 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	Answer the following: a) Compare NAD83(CSRS) with ITRF by listing two important differences between them (stating their full names). b) Compare CGVD28 with CGVD2013 by listing two differences between them (providing how it is realized, type of height system, reference surface for defining them, simple method of transforming from one to another). c) Explain two important differences between UT1 and UTC (stating their full names and the main properties distinguishing them). d) Briefly describe the term Tissot indicatrix using equivalent projection as an example. e) What is MTM (3TM) projection? Provide three important differences between UTM and MTM projections. f) Canada is planning to migrate from NAD83(CSRS) to the proposed NATRF2022 in parallel with the U.S. in 2025. Provide the full name of NATRF2022 and explain two important differences between it and NAD83 (CSRS). g) Explain *spatial reference system* and its reference frame (including how they are defined). h) Explain *combined scale factor* and how it is used. i) Explain meridian convergence and provide the expected magnitude of the meridian convergence of UTM projection at the UTM central meridian and the zone boundaries along the Equator. j) Briefly describe the term conformal map with regards to what it preserves and what it does not.	4 4 4 4 4 4 3 3 3 3	
2.	The map projection equations relating map projection coordinates (x, y) with the corresponding geographic coordinates (ϕ, λ) can be given as $x = R\lambda$ $\quad\quad$ $y = R\sin\phi$ where R is the mean radius of the spherical earth. Determine (showing all of your mathematical derivations and substitutions of values) if the projection is equal area using two different approaches (the conclusions of the two approaches must be identical).	10	
3.	a) A 3TM zone (with False Easting of 304,800 m and the scale factor of central meridian of 0.99990) and a UTM zone have the same central meridian. Calculate the UTM coordinates of the point whose 3TM map coordinates are X = 274,800.000 m, Y = 5,500,000.000 m. b) Determine the longitude coordinates (along the Equator) of the points where the scale factor distortion is minimal in UTM Zone 10N projection. What is that scale factor distortion?	5 5	

4.	On an UTM projection, point A is located on the eastern secant line while point B is located on a line where the scale factor is 1.0001 towards the eastern boundary. If the UTM plane bearing of line AB is 45º30´10.5", determine the grid bearing of the line (assuming the Gaussian mean radius of the earth in the region is 6,382,129.599 m).	20	
5.	a) Using well labelled sketches only, illustrate the Transverse Mercator (TM) and the Mercator projections in the Northern hemisphere; give one sketch for the TM projection and the other sketch for the Mercator projection. The sketches must show the projections of the Equator, Central Meridian, parallels and meridians with appropriate relationship between lines of the graticule clearly illustrated (with parallels and meridians at least six lines each). b) Describe how Tissot indicatrix will vary in sizes and shapes (specifying numerical values for parameters of the indicatrices, where possible) along the Equator and along the Central Meridians on the Transverse Mercator and Mercator projections.	16 8	
		100	

Some potentially useful formulae are given as follows:

$$T - t = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - x_0$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i.

UTM average line scale factor, $\bar{k}_{UTM} = k_0\left[1 + \frac{x_u^2}{6R_m^2}\left(1 + \frac{x_u^2}{36R_m^2}\right)\right]$;

where $x_i = x_i^{UTM} - x_0$; $x_u^2 = x_1^2 + x_1 x_2 + x_2^2$

UTM point scale factor, $k_{UTM} = k_0\left[1 + \frac{\Delta x^2}{2R_m^2}\right]$, where $\Delta x = x^{UTM} - x_0$

$$k_{UTM} = k_0\left[1 + \frac{L^2}{2}\cos^2\phi\right]$$

k_0 is scale factor of Central Meridian and x_0 is the False easting value (or 500 000 m) $L = (\lambda - \lambda_0)$ (in radians) for a given longitude λ; and λ_0 is the longitude of the central meridian.

Grid convergence, $\gamma = L\left(1 + \frac{L^2}{3}(1 + 3\eta^2)\cos^2\phi\right)\sin\phi$

where $\eta^2 = e'^2\cos^2\phi$; $e'^2 = 0.006739496780$; $L = (\lambda - \lambda_0)$ (in radians); λ_0 is the longitude of the central meridian; and ϕ is the latitude of the given point.

Geodetic bearing: $\alpha = t + \gamma + (T - t)$

$$Sf = \frac{R_m}{R_m + H_m}$$

Transformation Formulas:

$$X_{(target)} = k_{0(target)} X_G + X_{0(target)}$$
$$Y_{(target)} = k_{0(target)} Y_G$$
$$X_G = \frac{\left[X_{(original)} - X_{0(original)} \right]}{k_{0(original)}}$$
$$Y_G = \frac{Y_{(original)}}{k_{0(original)}}$$

ITRF:

$$\mathbf{r}(\mathrm{t}) = \mathbf{r}_0 + \dot{\mathbf{r}}(\mathrm{t} - \mathrm{t}_0)$$

where $\mathbf{r}_0$ and $\dot{\mathbf{r}}$ are the position and velocity respectively at $\mathbf{t}_0$.

Distortion Formulas:

Given: $X = f(\phi, \lambda)$ $\qquad$ $Y = g(\phi, \lambda)$

$$E = f_\phi^2 + g_\phi^2 \qquad F = f_\phi f_\lambda + g_\phi g_\lambda \qquad G = f_\lambda^2 + g_\lambda^2$$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2}\ ;\ m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2 \cos^2 \phi}\ ;\ p = \frac{2\left(f_\phi f_\lambda + g_\phi g_\lambda\right)}{R^2 \cos \phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A_p' \ ; \qquad f_\phi = \frac{\partial f(\phi,\lambda)}{\partial \phi}$$

$$\sin A_p' = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{\left(f_\lambda g_\phi - f_\phi g_\lambda\right)^2 + \left(f_\phi f_\lambda + g_\phi g_\lambda\right)^2}}$$

Equivalency Condition: $f_\lambda g_\phi - f_\phi g_\lambda = \pm R^2 \cos \phi$

Cauchy-Riemann Equations: $f_\phi = -\frac{1}{\cos \phi} g_\lambda$; $\qquad$ $g_\phi = \frac{1}{\cos \phi} f_\lambda$