CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS **March 2023**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 5 questions on 3 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	In a certain boundary retracement project in a Province in Canada, the preliminary surveys consisted of two independent measurement schemes: one made using GPS, producing geocentric coordinates; and the other by conventional traversing based on UTM system. The independently estimated coordinate data are to be used to compute inverses for the azimuths and distances for the final lines to be cut. A series of astronomic observations were also taken at each of the points that had been established for comparison with the azimuths from these two methods. Answer the following:		
	a) Describe the coordinate systems (GPS and UTM coordinate systems) involved in the two measurement schemes (defining their origins and directions of axes).	7	
	b) Clearly explain how the computed inverses for the azimuths will differ from the corresponding astronomic azimuths (your explanation must demonstrate the understanding of the three azimuths involved with the indication of the directions of north in each case).	5	
	c) Describe how the inverses for the distances in the two measurement schemes will compare with each other (stating the distance types and how to transform from one type to another).	3	
	d) Assuming that a total station's mark-to-mark distance and angle measurements are made at each of the network points and used directly with the corresponding astronomic azimuths to produce another set of three-dimensional coordinate data, describe the (total station) coordinate system most likely established by this process (defining the origin and the directions of axes of the coordinate system).	4	
	e) Discuss one important use of reference ellipsoid in each of the following coordinate systems: UTM coordinate system, GPS coordinate system, and total station coordinate system (including in your discussion one important disadvantage of each system).	6	
2.	a) A 3TM zone (with False Easting of 304,800 m and the scale factor of central meridian of 0.99990) and a UTM zone have the same central meridian. Calculate the UTM coordinates of the point whose 3TM map coordinates are X = 274,800.000 m, Y = 5,500,000.000 m.	5	
	b) Determine the longitude coordinates (along the Equator) of the points where the scale factor distortion is minimal in UTM Zone 10N projection. What is that scale factor distortion?	5	
	c) Given the general scale factors along the meridians and the parallels for a map projection system as $1/\cos\phi$ and the Gaussian surface element as $f = 0$ (where ϕ is the latitude), determine mathematically if the map projection is conformal or equivalent and show that both distortion properties cannot be satisfied at the same time (showing all of your mathematical expressions and substitutions of values into them for full marks).	5	

3.	a) Right Ascension system is an inertial reference system. Explain why it is an inertial system, and discuss how its curvilinear coordinates are measured (including their units and zero points of the coordinates). b) Discuss the differences between equivalent and orthomorphic projections and explain why one is preferred over the other by the surveyors (stating the one that is preferred). c) Explain the important uses of Cauchy-Riemann equations and developable surfaces in map projections. d) Explain the important differences between UT1 and UTC (stating their full names and the main properties distinguishing them).	5 4 3 4	
4.	On an UTM projection, point A is located on the eastern secant line while point B is located on a line where the scale factor is 1.0001 towards the eastern boundary. If the UTM plane bearing of line AB is 45°30′10.5", determine the grid bearing of the line (assuming the Gaussian mean radius of the earth in the region is 6,382,129.599 m).	20	
5.	a) Using well labelled sketches only, illustrate the Transverse Mercator (TM) and the Mercator projections in the Northern hemisphere; give one sketch for the TM projection and the other sketch for the Mercator projection. The sketches must show the projections of the Equator, Central Meridian, parallels and meridians with appropriate relationship between lines of the graticule clearly illustrated (with parallels and meridians at least six lines each). b) Describe how Tissot indicatrix will vary in sizes and shapes (specifying numerical values for parameters of the indicatrices, where possible) along the Equator and along the Central Meridians on the Transverse Mercator and Mercator projections.	16 8	
		100	

Some potentially useful formulae are given as follows:

$$T - t = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - x_0$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i.

UTM average line scale factor, $\bar{k}_{UTM} = k_0\left[1 + \frac{x_u^2}{6R_m^2}\left(1 + \frac{x_u^2}{36R_m^2}\right)\right]$;

where $x_i = x_i^{UTM} - x_0$; $x_u^2 = x_1^2 + x_1x_2 + x_2^2$

UTM point scale factor, $k_{UTM} = k_0\left[1 + \frac{\Delta x^2}{2R_m^2}\right]$, where $\Delta x = x^{UTM} - x_0$

$$k_{UTM} = k_0\left[1 + \frac{L^2}{2}\cos^2\phi\right]$$

k_0 is scale factor of Central Meridian and x_0 is the False easting value (or 500 000 m)

$L = (\lambda - \lambda_0)$ (in radians) for a given longitude λ; and λ_0 is the longitude of the central meridian.

Grid convergence, $\gamma = L\left(1+\frac{L^2}{3}\left(1+3\eta^2\right)\cos^2\phi\right)\sin\phi$

where $\eta^2 = e'^2\cos^2\phi$; $e'^2 = 0.006739496780$; $L = \left(\lambda - \lambda_0\right)$ (in radians); λ_0 is the longitude of the central meridian; and ϕ is the latitude of the given point.

Geodetic bearing: $\alpha = t + \gamma + (T - t)$

$$Sf = \frac{R_m}{R_m + H_m}$$

Transformation Formulas:

$$X_{(target)} = k_{0(target)} X_G + X_{0(target)}$$
$$Y_{(target)} = k_{0(target)} Y_G$$
$$X_G = \frac{\left[X_{(original)} - X_{0(original)}\right]}{k_{0(original)}}$$
$$Y_G = \frac{Y_{(original)}}{k_{0(original)}}$$

ITRF:

$$\mathbf{r}(t) = \mathbf{r}_0 + \dot{\mathbf{r}}\,(t - t_0)$$

where $\mathbf{r}_0$ and $\dot{\mathbf{r}}$ are the position and velocity respectively at $\mathbf{t}_0$.

Distortion Formulas:

Given: $X = f\left(\phi,\lambda\right)$ $\quad$ $Y = g\left(\phi,\lambda\right)$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2};\ m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2\cos^2\phi};\ p = \frac{2\left(f_\phi f_\lambda + g_\phi g_\lambda\right)}{R^2\cos\phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A_p';$$

$$\sin A_p' = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{\left(f_\lambda g_\phi - f_\phi g_\lambda\right)^2 + \left(f_\phi f_\lambda + g_\phi g_\lambda\right)^2}}$$

Equivalency Condition: $f_\lambda g_\phi - f_\phi g_\lambda = \pm R^2\cos\phi$

Cauchy-Riemann Equations: $f_\phi = -\frac{1}{\cos\phi} g_\lambda$; $\qquad g_\phi = \frac{1}{\cos\phi} f_\lambda$