CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS **March 2022**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 5 questions on 3 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	a) Right Ascension system is an inertial reference system. Explain why it is an inertial system, and discuss how its curvilinear coordinates are measured (including their units and zero points of the coordinates). b) Explain one important difference between the International Terrestrial Reference system and the Instantaneous terrestrial system. c) Describe the origin and coordinate axes of the coordinate system that is commonly used for satellite positioning at each specific instant of time the satellite is observed and explain four of the important parameters needed to convert coordinates in this coordinate system into geocentric coordinate system. d) Describe the parameters of 14-parameter transformation, and explain the purpose of the transformation. e) Explain the important differences between UT1 and UTC (stating their full names and what distinguishes them).	5 2 8 4 4	
2.	On an UTM projection, point A is located on the eastern secant line while point B is located on a line where the scale factor is 1.0001 towards the eastern boundary. If the UTM plane bearing of line AB is 45º30′10.5", determine the grid bearing of the line (assuming the Gaussian mean radius of the earth in the region is 6,382,129.599 m).	20	
3.	a) Discuss briefly the concept of Tissot indicatrix and describe the characteristics of Tissot indicatrices (with consideration for their sizes and shapes) as they will appear when drawn along the Equator and along the Central Meridian of each of the following map projections: Lambert Conformal Conic (one standard parallel), Mercator and UTM (identifying also the regions with perfect indicatrix in each case). b) Clearly explain why the plane bearing of a line P-Q in a Stereographic double projection and the plane bearing of the same line in a UTM projection cannot necessarily be compared directly as a check on the correct orientation of the line. Suggest, with justification, what might be more appropriate bearings to compare. c) Describe the variation of grid convergence in conic projections and explain why you would use conic projections for mapping Canada land mass.	15 4 3	
4.	The map projection equations relating map projection coordinates (x, y) with the corresponding geographic coordinates (ϕ, λ) can be given as $x = R\lambda$ $\qquad$ $y = R\sin\phi$ where R is the mean radius of the spherical earth. Determine (showing all of your mathematical derivations and substitutions of values) if the projection is equal area using two different approaches (the conclusions of the two approaches must be identical).	10	

5.	a) Using well labelled sketches only, illustrate the Transverse Mercator (TM) and the Polar Stereographic projections in the Northern hemisphere; give one sketch for the Transverse Mercator projection and the other sketch for the Polar Stereographic projection. The sketches must show the projections of the Equator, Central Meridian, parallels and meridians with appropriate relationship between lines of the graticule clearly illustrated. b) Explain two specific conditions that the TM projection must satisfy along the Central Meridian with regard to the *x* and *y* projection coordinates. c) What are the ellipsoidal (latitude and longitude) coordinates of the points where the meridian convergence values are minimal and maximal in a TM projection (Northern hemisphere) having a zone width of 6° and a Central Meridian at 123°W? Calculate the meridian convergence values corresponding to those points (assuming the Northern boundary of the zone is at 84°N).	16 3 6	
		100	

Some potentially useful formulae are given as follows:

$$T - t = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - x_0$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point *i*.

UTM average line scale factor, $\bar{k}_{UTM} = k_0\left[1 + \frac{x_u^2}{6R_m^2}\left(1 + \frac{x_u^2}{36R_m^2}\right)\right]$;

where $x_i = x_i^{UTM} - x_0$; $x_u^2 = x_1^2 + x_1x_2 + x_2^2$

UTM point scale factor, $k_{UTM} = k_0\left[1 + \frac{\Delta x^2}{2R_m^2}\right]$, where $\Delta x = x^{UTM} - x_0$

$$k_{UTM} = k_0\left[1 + \frac{L^2}{2}\cos^2\phi\right]$$

k_0 is scale factor of Central Meridian and x_0 is the False easting value (or 500 000 m) $L = (\lambda - \lambda_0)$ (in radians) for a given longitude λ; and λ_0 is the longitude of the central meridian.

Grid convergence, $\gamma = L\left(1 + \frac{L^2}{3}\left(1 + 3\eta^2\right)\cos^2\phi\right)\sin\phi$

where $\eta^2 = e'^2\cos^2\phi$; $e'^2 = 0.006739496780$; $L = (\lambda - \lambda_0)$ (in radians); λ_0 is the longitude of the central meridian; and ϕ is the latitude of the given point.

Geodetic bearing: $\alpha = t + \gamma + (T - t)$

$$Sf = \frac{R_m}{R_m + H_m}$$

<u>Transformation Formulas:</u>

$$X_{(target)} = k_{0(target)}X_G + X_{0(target)}$$

$$Y_{(target)} = k_{0(target)} Y_G$$

$$X_G = \frac{\left[X_{(original)} - X_{0(original)}\right]}{k_{0(original)}}$$

$$Y_G = \frac{Y_{(original)}}{k_{0(original)}}$$

ITRF:

$$\mathbf{r}(t) = \mathbf{r}_0 + \dot{\mathbf{r}}\,(t - t_0)$$

where $\mathbf{r}_0$ and $\dot{\mathbf{r}}$ are the position and velocity respectively at $\mathbf{t}_0$.

Distortion Formulas:

Given: $X = f(\phi, \lambda)$ $\qquad$ $Y = g(\phi, \lambda)$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2}\ ;\ m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2 \cos^2\phi}\ ;\ p = \frac{2\left(f_\phi f_\lambda + g_\phi g_\lambda\right)}{R^2 \cos\phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A_p^{'}\ ;$$

$$\sin A_p^{'} = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{\left(f_\lambda g_\phi - f_\phi g_\lambda\right)^2 + \left(f_\phi f_\lambda + g_\phi g_\lambda\right)^2}}$$

Equivalency Condition: $f_\lambda g_\phi - f_\phi g_\lambda = \pm R^2 \cos\phi$

Cauchy-Riemann Equations: $f_\phi = -\dfrac{1}{\cos\phi} g_\lambda\ ;$ $\qquad$ $g_\phi = \dfrac{1}{\cos\phi} f_\lambda$