## CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

## C-3 ADVANCED SURVEYING

October 2023

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to $\mathbf{2}$ more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 5 questions on 5 pages. Marks
Q. No

Time: 3 hours
Value Earned

| 1. | a) A leveling instrument that has not been used for over 20 years is to be used for a survey project. The manufacturer claims, following DIN 18723 (or ISO 17123 , now), that the equipment has a standard deviation of $\pm 0.2 \mathrm{~mm}$ over 1km double-run leveling. Since there is no record of any testing or calibration of this particular instrument, answer the following: <br> (i) Explain (with justification) six important Quality Assurance (QA) /Quality Control (QC) measures, and the field procedure (including the number and types of measurements made in the field) that you would recommend following in order to determine whether the level is capable of behaving as the manufacturer claimed. <br> (ii) Explain four quantities that will be determined from the processing of the measurements and fully discuss the statistical tests that will be performed on some of the quantities in order to determine whether the level is capable of behaving as the manufacturer claimed. <br> b) Provide four important Quality Assurance (QA) /Quality Control (QC) measures (including their purposes) that you would recommend following for successful calibration of an electromagnetic distance measurement (EDM) instrument. | 10 8 8 |
| :---: | :---: | :---: |
| 2. | A traverse is to be measured around a rectangular city block (ABCD) of 100 m by 200 m . The two 200 m sides are relatively flat while the other two have slopes of $20 \%$. The equipment (total station or targets) would be set up on tripods with height of instrument or height of target of 1.600 m . Since this survey may extend over more than one session and only ground mark points will be occupied, forced centering cannot be assumed. The maximum allowable angular misclosure (at 99\% confidence) of the traverse is $15^{\prime \prime}$. With consideration for the effects of centering, leveling, pointing, and reading, numerically determine the conditions under which the misclosure will be satisfied using Leica TPS 802 total station instrument (assuming only the total station instrument is to be re-centered and re-leveled between sets). The Leica TPS 802 specifications are: standard deviation of an angle measurement (ISO 17123-3) is $2^{\prime \prime}$; and compensator setting accuracy is $1^{\prime \prime}$. | 20 |
| 3. | a) What is a correlation survey with regards to a tunneling project with access only through vertical shafts? Explain two important reasons why it is necessary. <br> b) The transfer of elevation from the surface benchmarks via a single shaft to underground stations can be done using total station equipment or steel tape. Briefly explain three important disadvantages of using tape and three important disadvantages of using total station in the elevation transfer. | 5 8 |


| 4. | a) In a deformation survey, the datum-independent displacement vector $(\hat{d})$ and the corresponding cofactor matrix ( $Q_{\hat{d}}$ ) for a monitored point as follows: $\hat{d}=\left[\begin{array}{l} d E \\ d N \end{array}\right]=\left[\begin{array}{l} -0.013 m \\ -0.006 m \end{array}\right] \quad Q_{\hat{d}}=\left[\begin{array}{ll} 2.0323 E-5 & 1.0665 E-6 \\ 1.0665 E-6 & 1.2796 E-5 \end{array}\right] m^{2}$ <br> with the pooled variance factor as 0.518 , the number of degrees of freedom for the pooled variance as 28 and the number of network points being considered for deformation analysis as 5 . Determine the statistical significance of the displacement at this point at the significance level, $\alpha=5 \%$, assuming the a priori variance factor of unit weight is well known, and explain how you would have checked if the measurements in the two epochs involved are compatible before the deformation analysis. <br> b) Discuss how a surveyor can use geodetic level as a geotechnical tiltmeter in deformation monitoring (explaining the procedure, the observables, formulae involved, and one possible advantage of the procedure over tiltmeter). | 8 6 |  |
| :---: | :---: | :---: | :---: |
| 5. | a) You are to perform Second Order Design (SOD) for horizontal positioning using the process of simulation. The relative positioning tolerance (limit on relative error ellipses at $95 \%$ confidence level) is to be 0.010 m and the potential observables will be angles and distances. Explain step-by-step and logically your whole plan on how to perform the simulation. Your plan must be complete enough for a computer programmer to use in developing a software application for the network design (do not be tempted to say, "Just use Pre-analysis or Design software"). You must provide the following for full marks: interpretation of the given tolerance, how to select potential geometry; input to the design and their underlying equations; design equations; general computational steps and appropriate equations and matrix expressions used at each step of the process (refer to the attached formula sheets for relevant equations and matrix expressions). Marks will be awarded depending on your demonstration of full understanding of the process involved and how much relevant details and formulae are logically provided. <br> b) Discuss in detail the purpose of GNSS validation (including elements validated) and describe one important statistical test (including necessary statistical formulae with symbols well defined, and the expected degrees of freedom and confidence level) that may be used to check the external accuracy of the validation. | 20 |  |
|  |  | 100 |  |

Some potentially useful formulae are given as follows:

$$
\begin{aligned}
& v=\frac{Z_{I}+Z_{I I}-360}{2} \\
& \bar{z}=\frac{Z_{I}+\left(360-Z_{I I}\right)}{2} \\
& \frac{c}{\sin (z)}=\frac{H z_{I}-\left(H z_{I I}-180\right)}{2} \quad \frac{t}{\tan (z)}+\frac{c}{\sin (z)}=\frac{H z_{I}-\left(H z_{I I}-180\right)}{2} \\
& \text { Corrected direction }=\text { Measured direction }-\frac{(N R-N L) \times v^{\prime \prime}}{2 \tan z} \\
& i_{v}=z-z^{\prime} \quad \text { or } \quad i_{v}=i \cos \alpha ; \quad i_{T}=H z-H z^{\prime} \text { or } \quad i_{T}=\frac{i \sin \alpha}{\tan z} \\
& \text { Deformation: } \ell_{2}-\ell_{1}+V=A d \text {; } \\
& d=\hat{x}_{2}-\hat{x}_{1}
\end{aligned}
$$

$$
\begin{aligned}
& F_{c}=\frac{\hat{a}^{T} Q_{\bar{d}}^{-1} \hat{d}}{\hat{\sigma}_{0}^{2} u_{d}}<F\left(1-\alpha_{0}, u_{d}, d f_{p}\right) ; \quad F_{c}=\frac{\hat{d}^{T} Q_{\hat{d}}^{-1} \hat{d}}{\hat{\sigma}_{0}^{2} u_{d}}<\frac{\chi_{1-\alpha_{0}, d f=u_{d}}^{u_{d}}}{\alpha=\frac{\delta \Delta h}{s} \quad \text { where } \delta \Delta \mathrm{h}=\Delta \mathrm{h}_{12 t 2}-\Delta_{\mathrm{h} 12 \mathrm{t} 1} .} \\
& \sigma_{\alpha}=\frac{\sigma_{\delta \Delta h}}{s} \quad \text { where } \sigma_{\delta \Delta h}=\sqrt{\sigma_{\Delta h 1}^{2}+\sigma_{\Delta h 2}^{2}}
\end{aligned}
$$

EDM:

$$
\begin{aligned}
& n_{a}=1+\frac{\left(n_{g}-1\right) 273.16 p}{(273.16+t) 1013.25} \quad\left(\text { for } p \text { in } \mathrm{mb} \text { and } t \text { in }{ }^{\circ} \mathrm{C}\right) \\
& N=(n-1) \times 10^{6} \quad \delta^{\prime}=\left(N_{\text {REF }}-N_{a}\right) d^{\prime} \times 10^{-6}
\end{aligned}
$$

Standard pressure: 760 mmHg or $1013.25 \mathrm{mb} ; 0^{\circ} \mathrm{C}$ or 273.15 K

$$
\hat{C}=\frac{M-\left(m_{1}+m_{2}+m_{3}+m_{4}+\ldots+m_{n}\right)}{n-1}
$$

Statistics:

$$
\begin{array}{rll}
|\Delta| & =\sigma_{\Delta} \sqrt{\chi_{1-\alpha, d f}^{2}} & |\Delta| \leq z_{1-\alpha / 2} \sigma_{\Delta} \\
y & =d \hat{x}^{T} C_{\hat{x}}^{-1} d \hat{x} \quad \chi_{\frac{\alpha}{2}, d f}^{2} \leq \frac{(d f) s^{2}}{\sigma^{2}} \leq \chi_{1-\frac{\alpha}{2}, d f}^{2} & F_{1-\frac{\alpha}{2}, d f_{1}, d f_{2}} \leq \frac{s_{01}^{2}}{s_{02}^{2}} \leq F_{\frac{\alpha}{2}, d f f_{1}, d f_{2}} \\
y & <\chi_{u, 1-\alpha}^{2} \quad a_{(1-\alpha) 100 \%}=a_{s t} \sqrt{\chi_{1-\alpha, d f}^{2}} \quad \text { or } & a_{(1-\alpha) 100 \%}=a_{s t} \sqrt{2 F_{1-\alpha, d f, d f 2}}
\end{array}
$$

## Error propagation:

$$
\begin{aligned}
& \sigma_{d p}=\frac{\sigma_{p}}{\sqrt{2 n}} \quad \sigma_{d p}=\frac{60}{M} \quad \sigma_{\theta P}=\frac{\sigma_{P}}{\sqrt{n}} \quad \sigma_{d r}=\frac{\sigma_{r}}{\sqrt{2 n}} \quad \sigma_{d r}=2.5 \operatorname{div} \quad \sigma_{\theta r}=\frac{\sigma_{r}}{\sqrt{n}} \\
& \sigma_{L}=\sigma_{v} \cot z, \quad \sigma_{v}=0.2 v^{\prime \prime} \quad \sigma_{\mathrm{r}}=2.5 \mathrm{~d}^{\prime \prime} \quad \sigma_{\theta L}=\sigma_{v} \sqrt{\cot ^{2}\left(Z_{b}\right)+\cot ^{2}\left(Z_{f}\right)} \\
& \sigma_{i}=\frac{\left(206265^{\prime \prime}\right) \sigma_{c 3}}{S_{1}} \quad \sigma_{t}=\frac{\left(206265^{\prime \prime}\right) \sigma_{c 1}}{S_{1}} \quad \sigma_{d c}=\frac{206265}{s} \sqrt{\sigma_{c 3}^{2}+\sigma_{c 1}^{2}} \\
& \sigma_{c}= \pm 0.5 \mathrm{~mm} / \mathrm{m} \times \mathrm{HI}(\mathrm{~m}) \quad \sigma_{c}= \pm 0.1 \mathrm{~mm} \quad \sigma_{c}= \pm 0.1 \mathrm{~mm} / \mathrm{m} \times \mathrm{HI}(\mathrm{~m}) \\
& \sigma_{\theta i}=\left(206265^{\prime \prime}\right) \sigma_{c 3} \sqrt{\left[\frac{S_{1}^{2}+S_{2}^{2}-2 S_{1} S_{2} \cos \theta}{S_{1}^{2} S_{2}^{2}}\right]} \\
& \sigma_{P}=\frac{45}{206265 \times M} S ; \quad \sigma_{L}=\left(\frac{\sigma_{v}}{206265}\right) S ; \quad \sigma_{r}=\frac{\ell}{2}\left(\frac{v_{r}}{206265}\right)^{2} \\
& \sigma_{d}=\frac{S}{2 R} \sigma_{k_{h}} \quad \sigma_{r e f}=\frac{S}{2 R} \sigma_{k_{v}} \\
& \ell=f(x) \quad C_{\hat{x}}=\sigma_{0}^{2}\left(A^{T} P A\right)^{-1} \quad \mathrm{P}=Q^{-1} \\
& s_{\Delta x}^{2}=s_{x_{1}}^{2}+s_{x_{2}}^{2}-2 s_{x_{1} x_{2}} \quad s_{\Delta x \Delta y}=s_{x_{1} y_{1}}+s_{x_{2} y_{2}}-s_{x_{1} y_{2}}-s_{y_{1} x_{2}} \quad s_{\Delta y}^{2}=s_{y_{1}}^{2}+s_{y_{2}}^{2}-2 s_{y_{1} y_{2}} \\
& \lambda_{1}=\frac{1}{2}\left(s_{\Delta x}^{2}+s_{\Delta y}^{2}+R\right) \quad \lambda_{2}=\frac{1}{2}\left(s_{\Delta x}^{2}+s_{\Delta y}^{2}-R\right) \quad R=\left[\left(s_{\Delta x}^{2}-s_{\Delta y}^{2}\right)^{2}+4 s_{\Delta x \Delta y}^{2}\right]^{1 / 2} \\
& a_{s}=\sqrt{\lambda_{1}} \quad b_{s}=\sqrt{\lambda_{2}} \\
& a_{95}=k_{95} a_{s} \\
& b_{95}=k_{95} b_{s}
\end{aligned}
$$

$$
k_{95}=\sqrt{\chi_{d f, 1-\alpha}^{2}} \quad \beta=\arctan \left(\frac{s_{\Delta x \Delta y}}{\lambda_{1}-s_{\Delta x}^{2}}\right)
$$

## Map projection and Reductions:

Meridian convergence: $\gamma=\frac{d \tan \phi\left(1-e^{2} \sin ^{2} \phi\right)^{1 / 2}}{a} \quad \mathrm{a}=6378137 \mathrm{~m} ; \mathrm{e}=0.081819191$ or $\gamma=L\left(1+\frac{L^{2}}{3}\left(1+3 \eta^{2}\right) \cos ^{2} \phi\right) \sin \phi$
where $\eta^{2}=\mathrm{e}^{\prime 2} \cos ^{2} \phi ; e^{\prime 2}=0.006739496780 ; L=\left(\lambda-\lambda_{0}\right)$ (in radians); $\lambda_{0}$ is the longitude of the central meridian; and $\phi$ is the latitude of the given point.
$\alpha=A-\eta \tan \phi \quad$ where $-\eta \tan \phi$ is Laplace correction

## Horizontal Control Survey:

$\mathrm{a}=\mathrm{C}(\mathrm{d}+0.2) \mathrm{cm} \quad$ [where d is distance in $\mathrm{km} ; \mathrm{C}=2$ (First Order); $\mathrm{C}=5$ (Second Order)]

## Vertical Control survey:

$\pm 3 m m \sqrt{L} \quad \pm 4 m m \sqrt{L} \quad \pm 8 m m \sqrt{L} \quad \pm 24 m m \sqrt{L} \quad \pm 120 m m \sqrt{L}$

## Map Accuracy Standards:

$$
\begin{array}{ll}
\text { Accuracy }_{x}=S E \times \sqrt{\chi_{d f, 1-\alpha}^{2}} & \text { Accuracy }=S E \times \sqrt{\chi_{d f, 1-\alpha}^{2}} \\
\text { Accuracy } & =S E \times z_{1-\alpha / 2} \\
V M A S=C I / 2 \quad V M A S=S E \times z_{1-\alpha / 2} \\
\text { VMAS }=S E \times z_{1-\alpha / 2} & S E=R M S E
\end{array}
$$

Table 1: Normal Distribution table (upper tail area):

| $\alpha$ | 0.001 | 0.002 | 0.003 | 0.004 | 0.005 | 0.01 | 0.025 | 0.05 | 0.10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Z}_{\alpha}$ | 3.09 | 2.88 | 2.75 | 2.65 | 2.58 | 2.33 | 1.96 | 1.64 | 1.28 |

Table 2: Table for Student-t distribution ( $\alpha$ is upper tail area)

|  | $\mathrm{t}_{\alpha}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Degree of freedom | $\mathrm{t}_{0.10}$ | $\mathrm{t}_{0.05}$ | $\mathrm{t}_{0.025}$ | $\mathrm{t}_{0.01}$ |
| 1 | 3.08 | 6.31 | 12.7 | 31.8 |
| 2 | 1.89 | 2.92 | 4.30 | 6.96 |
| 2 | 1.64 | 2.35 | 3.18 | 4.54 |
| 3 | 1.53 | 2.13 | 2.78 | 3.75 |
| 4 | 1.48 | 2.01 | 2.57 | 3.36 |
| 5 | 1.49 | 1.94 | 2.45 | 3.14 |
| 6 | 1.363 | 1.796 | 2.201 | 2.718 |
| 11 | 1.356 | 1.782 | 2.179 | 2.681 |
| 12 | 1.350 | 1.771 | 2.160 | 2.650 |
| 13 | 1.345 | 1.761 | 2.145 | 2.624 |
| 14 | 1.341 | 1.753 | 2.131 | 2.602 |
| 15 |  |  |  |  |

Table 3: Chi-Square Distribution table (lower tail area)

| $\alpha$ | 0.025 | 0.05 | 0.10 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Degrees of <br> freedom |  |  |  |  |  |  |  |  |
| 1 | 0.001 | 0.004 | 0.016 | 2.705 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| 11 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 |
| 12 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 |
| 13 | 5.009 | 5.892 | 7.041 | 19.811 | 22.362 | 24.736 | 27.688 | 29.819 |
| 14 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 |
| 28 | 15.308 | 16.928 | 18.939 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 |

