CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-3 ADVANCED SURVEYING **March 2024**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 5 questions on 5 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	a) Two survey crews A and B measured the length of a horizontal baseline using the same EDM instrument (with precision ± 2 mm ± 2 ppm and the reference refractivity as 281.949). Crew A measured the whole baseline and obtained the overall length of the baseline as 799.935 m (but uncorrected for meteorological condition with refractivity value of 300.000). Crew B measured the baseline in two equal sections (with each section measured independently) and obtained the meteorologically corrected overall length of the baseline as 799.931 m. Assume that each of the crews were able to center their instrument to an accuracy of 1.0 mm and their target to 1.0 mm. Determine if there is any significant difference (at 95% confidence level) between the two lengths obtained by crews A and B (based on their corrected values) and clearly explain the sources of errors accounted for in the EDM precision specification.	11	
	b) What quantities are estimated and statistically analyzed in EDM calibration procedure? Explain (with justification) what statistical tests are done for each quantity estimated.	5	
	c) The standard deviation of Leica DNA03 digital level with invar staff is specified by the manufacturer as 0.3 mm/km double-run according to ISO17123-2 standard. The least squares adjusted experimental standard deviation of elevation difference (over 60 m) in the calibration process is 0.18 mm with the degrees of freedom of 15. Test statistically at 95% confidence level if this standard deviation is acceptable (not greater than the manufacturer's specification for the level). (For full marks, your answer must clearly present intermediate formulae used and error propagations, null and alternative hypotheses, statistic, test and conclusion.)	8	
2.	A traverse is to be measured around a rectangular city block (ABCD) of 100 m by 200 m. The two 200 m sides are relatively flat while the other two have slopes of 20%. The equipment (total station or targets) would be set up on tripods with height of instrument or height of target of 1.600 m. Since this survey may extend over more than one session and only ground mark points will be occupied, forced centering cannot be assumed. The maximum allowable angular misclosure (at 99% confidence) of the traverse is 15″. With consideration for the effects of centering, leveling, pointing, and reading, numerically determine the conditions under which the misclosure will be satisfied using Leica TPS 802 total station instrument (assuming only the total station instrument is to be re-centered and re-leveled between sets). The Leica TPS 802 specifications are: standard deviation of an angle measurement (ISO 17123-3) is 2″; and compensator setting accuracy is 1″.	20	

3.	a) What is a correlation survey with regards to a tunneling project with access only through vertical shafts? Explain two important reasons why it is necessary. b) The transfer of elevation from the surface benchmarks via a single shaft to underground stations can be done using total station equipment or steel tape. Briefly explain three important disadvantages of using tape and three important disadvantages of using total station in the elevation transfer.	5 8	
4.	a) In a deformation survey, the datum-independent displacement vector ($\hat{d}$) and the corresponding cofactor matrix ($Q_{\hat{d}}$) for a monitored point as follows: $\hat{d} = \begin{bmatrix} dE \\ dN \end{bmatrix} = \begin{bmatrix} -0.013m \\ -0.006m \end{bmatrix} \quad Q_{\hat{d}} = \begin{bmatrix} 2.0323E-5 & 1.0665E-6 \\ 1.0665E-6 & 1.2796E-5 \end{bmatrix} m^2$ with the pooled variance factor as 0.518, the number of degrees of freedom for the pooled variance as 28 and the number of network points being considered for deformation analysis as 5. Determine the statistical significance of the displacement at this point at the significance level, α = 5%, assuming the a priori variance factor of unit weight is well known, and explain how you would have checked if the measurements in the two epochs involved are compatible before the deformation analysis. b) Discuss how a surveyor can use geodetic level as a geotechnical tiltmeter in deformation monitoring (explaining the procedure, the observables, formulae involved, and one possible advantage of the procedure over tiltmeter).	8 6	
5.	a) You are to perform Second Order Design (SOD) for horizontal positioning using the process of simulation. The relative positioning tolerance (limit on relative error ellipses at 95% confidence level) is to be 0.010 m and the potential observables will be angles and distances. Explain step-by-step and logically your whole plan on how to perform the simulation. Your plan must be complete enough for a computer programmer to use in developing a software application for the network design (do not be tempted to say, “Just use Pre-analysis or Design software”). You must provide the following for full marks: interpretation of the given tolerance, how to select potential geometry; input to the design and their underlying equations; design equations; general computational steps and appropriate equations and matrix expressions used at each step of the process (refer to the attached formula sheets for relevant equations and matrix expressions). Marks will be awarded depending on your demonstration of full understanding of the process involved and how much relevant details and formulae are logically provided. b) Discuss in detail the purpose of GNSS validation (including elements validated) and describe one important statistical test (including necessary statistical formulae with symbols well defined, and the expected degrees of freedom and confidence level) that may be used to check the external accuracy of the validation.	20 9	
		100	

Some potentially useful formulae are given as follows:

$$v = \frac{Z_I + Z_{II} - 360}{2} \qquad \bar{z} = \frac{Z_I + (360 - Z_{II})}{2}$$

$$\frac{c}{\sin(z)} = \frac{Hz_I - (Hz_{II} - 180)}{2} \qquad \frac{t}{\tan(z)} + \frac{c}{\sin(z)} = \frac{Hz_I - (Hz_{II} - 180)}{2}$$

Corrected direction = Measured direction – $\frac{(NR-NL)\times v''}{2\tan z}$

$i_v = z - z'$ or $i_v = i\cos\alpha$; $i_T = Hz - Hz'$ or $i_T = \frac{i\sin\alpha}{\tan z}$

Deformation: $\ell_2 - \ell_1 + V = Ad$; $d = \hat{x}_2 - \hat{x}_1$

$F_c = \frac{\hat{d}^T Q_{\hat{d}}^{-1}\hat{d}}{\hat{\sigma}_0^2 u_d} < F(1-\alpha_0, u_d, df_p)$; $F_c = \frac{\hat{d}^T Q_{\hat{d}}^{-1}\hat{d}}{\hat{\sigma}_0^2 u_d} < \frac{\chi^2_{1-\alpha_0, df=u_d}}{u_d}$

$\alpha = \frac{\delta\Delta h}{s}$ where $\delta\Delta h = \Delta h_{12t2} - \Delta h_{12\,t1}$.

$\sigma_\alpha = \frac{\sigma_{\delta\Delta h}}{s}$ where $\sigma_{\delta\Delta h} = \sqrt{\sigma^2_{\Delta h1} + \sigma^2_{\Delta h2}}$

EDM: $n_a = 1 + \frac{(n_g - 1)273.16p}{(273.16+t)1013.25}$ (for p in mb and t in °C)

$N = (n-1)\times 10^6$ $\delta' = (N_{REF} - N_a)d'\times 10^{-6}$

Standard pressure: 760 mmHg or 1013.25 mb; 0°C or 273.15 K

$$\hat{C} = \frac{M - (m_1 + m_2 + m_3 + m_4 + ... + m_n)}{n-1}$$

Statistics:

$|\Delta| = \sigma_\Delta\sqrt{\chi^2_{1-\alpha,df}}$ $|\Delta| \le z_{1-\alpha/2}\sigma_\Delta$ $|\Delta| \le t_{df,1-\alpha/2}\sigma_\Delta$ $\hat{\sigma} \le \sqrt{\frac{\chi^2_{1-\alpha,df}(\sigma^2)}{df}}$

$y = d\hat{x}^T C_{\hat{x}}^{-1} d\hat{x}$ $\chi^2_{\frac{\alpha}{2},df} \le \frac{(df)s^2}{\sigma^2} \le \chi^2_{1-\frac{\alpha}{2},df}$ $F_{1-\frac{\alpha}{2},df_1,df_2} \le \frac{s_{01}^2}{s_{02}^2} \le F_{\frac{\alpha}{2},df_1,df_2}$

$y < \chi^2_{u,1-\alpha}$ $a_{(1-\alpha)100\%} = a_{st}\sqrt{\chi^2_{1-\alpha,df}}$ or $a_{(1-\alpha)100\%} = a_{st}\sqrt{2F_{1-\alpha,df1,df2}}$

Error propagation: Leveling: $d\bar{h} = \frac{dh_f - dh_b}{2}$ $\sigma_{\Delta h(1km)} = \sigma_{\Delta h}\sqrt{m}$

$\Delta h_{1km} = \Delta h_1 + \Delta h_2 + \ldots + \Delta h_m$

$\sigma_{dp} = \frac{\sigma_p}{\sqrt{2n}}$ $\sigma_{dp} = \frac{60}{M}$ $\sigma_{\theta P} = \frac{\sigma_P}{\sqrt{n}}$ $\sigma_{dr} = \frac{\sigma_r}{\sqrt{2n}}$ $\sigma_{dr} = 2.5$ div $\sigma_{\theta r} = \frac{\sigma_r}{\sqrt{n}}$

$\sigma_L = \sigma_v \cot z$, $\sigma_v = 0.2v''$ $\sigma_r = 2.5d''$ $\sigma_{\theta L} = \sigma_v\sqrt{\cot^2(Z_b) + \cot^2(Z_f)}$

$\sigma_i = \frac{(206265'')\sigma_{c3}}{S_1}$ $\sigma_t = \frac{(206265'')\sigma_{c1}}{S_1}$ $\sigma_{dc} = \frac{206265}{S}\sqrt{\sigma_{c3}^2 + \sigma_{c1}^2}$

$\sigma_c = \pm 0.5mm$/m × HI (m) $\sigma_c = \pm 0.1$ mm $\sigma_c = \pm 0.1$ mm/m × HI (m)

$$\sigma_{\theta i} = (206265'')\sigma_{c3}\sqrt{\left[\frac{S_1^2 + S_2^2 - 2S_1S_2\cos\theta}{S_1^2 S_2^2}\right]}$$

$\sigma_P = \frac{45}{206265\times M}S$; $\sigma_L = \left(\frac{\sigma_v}{206265}\right)S$; $\sigma_r = \frac{\ell}{2}\left(\frac{v_r}{206265}\right)^2$

$\sigma_d = \frac{S}{2R}\sigma_{k_h}$ $\sigma_{ref} = \frac{S}{2R}\sigma_{k_V}$

$\ell = f(x)$ $\quad C_{\hat{x}} = \sigma_0^2 \left(A^T PA\right)^{-1}$ $\quad P = Q^{-1}$

$s_{\Delta x}^2 = s_{x_1}^2 + s_{x_2}^2 - 2s_{x_1 x_2}$ $\quad s_{\Delta x \Delta y} = s_{x_1 y_1} + s_{x_2 y_2} - s_{x_1 y_2} - s_{y_1 x_2}$ $\quad s_{\Delta y}^2 = s_{y_1}^2 + s_{y_2}^2 - 2s_{y_1 y_2}$

$\lambda_1 = \frac{1}{2}\left(s_{\Delta x}^2 + s_{\Delta y}^2 + R\right)$ $\quad \lambda_2 = \frac{1}{2}\left(s_{\Delta x}^2 + s_{\Delta y}^2 - R\right)$ $\quad R = \left[\left(s_{\Delta x}^2 - s_{\Delta y}^2\right)^2 + 4s_{\Delta x \Delta y}^2\right]^{1/2}$

$a_s = \sqrt{\lambda_1}$ $\quad b_s = \sqrt{\lambda_2}$ $\quad a_{95} = k_{95} a_s$ $\quad b_{95} = k_{95} b_s$

$k_{95} = \sqrt{\chi_{df,1-\alpha}^2}$ $\quad \beta = \arctan\left(\frac{s_{\Delta x \Delta y}}{\lambda_1 - s_{\Delta x}^2}\right)$

Map projection and Reductions:

Meridian convergence: $\gamma = \dfrac{d \tan\phi \left(1 - e^2 \sin^2 \phi\right)^{1/2}}{a}$ $\quad$ a = 6378137 m; e = 0.081819191

or $\gamma = L\left(1 + \frac{L^2}{3}\left(1 + 3\eta^2\right)\cos^2\phi\right)\sin\phi$

where $\eta^2 = e'^2 \cos^2\phi$; e'^2 =0.006739496780; $L = (\lambda - \lambda_0)$ (in radians); λ_0 is the longitude of the central meridian; and ϕ is the latitude of the given point.

$\alpha = A - \eta \tan\phi$ $\quad$ where $-\eta \tan\phi$ is Laplace correction

Horizontal Control Survey:

a = C(d +0.2) cm [where d is distance in km; C = 2 (First Order); C = 5 (Second Order)]

Vertical Control survey:

$\pm 3mm\sqrt{L}$ $\quad \pm 4mm\sqrt{L}$ $\quad \pm 8mm\sqrt{L}$ $\quad \pm 24mm\sqrt{L}$ $\quad \pm 120mm\sqrt{L}$

Map Accuracy Standards:

$Accuracy_x = SE \times \sqrt{\chi_{df,1-\alpha}^2}$ $\quad Accuracy_y = SE \times \sqrt{\chi_{df,1-\alpha}^2}$

$Accuracy_z = SE \times z_{1-\alpha/2}$ $\quad CMAS = SE \times z_{1-\alpha/2}$

$VMAS = CI/2$ $\quad VMAS = SE \times z_{1-\alpha/2}$ $\quad SE = RMSE$

Table 1: Normal Distribution table (upper tail area):

α	0.001	0.002	0.003	0.004	0.005	0.01	0.025	0.05	0.10
z_α	3.09	2.88	2.75	2.65	2.58	2.33	1.96	1.64	1.28

Table 2: Table for Student-t distribution (α is upper tail area)

	t_α			
Degree of freedom	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36
6	1.49	1.94	2.45	3.14
11	1.363	1.796	2.201	2.718
12	1.356	1.782	2.179	2.681
13	1.350	1.771	2.160	2.650
14	1.345	1.761	2.145	2.624
15	1.341	1.753	2.131	2.602

Table 3: Chi-Square Distribution table (lower tail area)

α	0.025	0.05	0.10	0.90	0.95	0.975	0.99	0.995
Degrees of freedom								
1	0.001	0.004	0.016	2.705	3.841	5.024	6.635	7.879
2	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
11	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	5.009	5.892	7.041	19.811	22.362	24.736	27.688	29.819
14	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
28	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993