CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-3 ADVANCED SURVEYING **March 2022**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 6 questions on 5 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	a) In a closed-loop horizontal traverse of 5 points, the following were calculated for each angle (not direction) of the traverse measured in one set: error due to centering of instrument is 2.5", error due to centering of target is 3.0", levelling error is 0.0", and the specified standard deviation of an angle measurement (according to ISO17123-3 standard) is 2.0". If each angle is now measured in three sets, with re-levelling and re-centering of the instrument (not the targets) between sets, determine if the maximum allowable angular misclosure of traverse of $10\sqrt{n}$ is achieved (where n is the number of traverse points, and interpreting the maximum allowable misclosure at 95% confidence level).	5	
	b) Using modern total stations [angular accuracy of $\pm 1''$ and a distance accuracy of ± 1 mm ± 1 ppm according to ISO Standards] has the potential for competing with precise differential levelling. Assuming the total station is used in a leap-frog trigonometric levelling procedure with imposed balanced maximum sight length of 50 m, determine numerically whether the levelling result will satisfy the First Order Canadian levelling specification (corresponding to 99% confidence level). [Let the average slope of the terrain (covered with the same material) be +1.5° and the standard deviation of height difference measurement between back sight and foresight targets at each setup be ±0.2 mm.]	16	
2.	a) Leica DNA03 digital level with stated accuracy of 0.3 mm/km double-run (ISO17123-2) was tested according to ISO17123-2 standards procedure and the least squares adjusted standard deviation of elevation difference in a setup (single-run) was estimated as 0.17 mm with the degrees of freedom of 14. Determine statistically if you will be confident in the manufacturer's claimed accuracy specification at 95% confidence level (clearly providing null and alternative hypotheses, test statistic, critical values, test and conclusion).	6	
	b) The system constant, z_0, of an EDM is to be determined without using calibration baseline with known lengths. Show mathematically (with clear and logical steps) how z_0 can be uniquely determined; and determine the standard deviation of z_0 assuming the standard deviation of measuring a distance by the EDM is ±3 mm and the centering of instrument and target can be done to an accuracy of 1 mm each.	6	
	c) GNSS system validation is one of the important GNSS field procedures that may be required prior to a GNSS control survey. Discuss briefly the key elements of the GNSS measurement validation procedure, purpose (including quantities to be validated), how often it is done, and explain a typical statistical test to check the external accuracy of the validation (including necessary statistical formulae with symbols well defined, and the expected degrees of freedom and confidence level).	13	

<table>
<tr><td>3.</td><td>a) A local plane coordinate system was established at the collar of a shaft at latitude of 59°30′N (with grid convergence equal to zero). At a depth of 2km, a tunnel runs approximately in a westerly direction from the shaft. An open traverse follows the tunnel with stations along one side. Gyro-azimuths, based on a well-calibrated Gyromat 3000 automated gyro station, were measured at regular intervals in order to "control" the orientation of the tunnel. The specified accuracy of an azimuth determination with the equipment and procedures is ± 3″ and the average deflection of the vertical along the tunnel is -5". Apart from the internal corrections that are automatically done by the gyro station to the gyro measurements, explain two other important systematic corrections, with justification and suggestions of their numerical values (with calculation steps), that should be applied to the azimuths, observed at 4 km (westerly) from the shaft in order to convert it to a grid azimuth in the surface coordinate system.
b) Clearly explain two major corrections usually applied to tape measurements in elevation transfer into an underground mine that are not normally applied when a tape is used to determine horizontal distances.</td><td>7

2</td><td></td></tr>
<tr><td>4.</td><td>a) In a deformation survey, the datum-independent displacement vector ($\hat{d}$) and the corresponding cofactor matrix ($Q_{\hat{d}}$) for a monitored point as follows:
$\hat{d} = \begin{bmatrix} dE \\ dN \end{bmatrix} = \begin{bmatrix} -0.013m \\ -0.006m \end{bmatrix} \quad Q_{\hat{d}} = \begin{bmatrix} 2.0323E-5 & 1.0665E-6 \\ 1.0665E-6 & 1.2796E-5 \end{bmatrix} m^2$
with the pooled variance factor as 0.518, the number of degrees of freedom for the pooled variance as 28 and the number of network points being considered for deformation analysis as 5. Determine the statistical significance of the displacement at this point at the significance level, α = 5%, assuming the a priori variance factor of unit weight is well known, and explain how you would have checked if the measurements in the two epochs involved are compatible before the deformation analysis.
b) Discuss how surveyor can use geodetic level as a geotechnical tiltmeter (explaining the observables, formulae involved, and one possible advantage over tiltmeter).</td><td>8

6</td><td></td></tr>
<tr><td>5.</td><td>You are required to carry out a topographic survey of a proposed construction project site of about 5 ha. The expected drawing scale is 1:1000; ground elevations are to be shot at 15 m grid spacing and the final drawings must be delivered with a 0.5 m contour interval and must meet the NMAS Standards. Some of the specifications for the typical map standards, such as NMAS and ASPRS are as follows:
• For NMAS, the horizontal tolerance is 0.82 mm for map scales larger than 1:20,000 and the elevation should be accurate to within one-half a contour interval.
• For ASPRS, the maximum allowable error (limiting RMSE) for X or Y coordinates of well-defined points for map of 1:1000 is 0.25 m; and the elevation should be accurate to within one-third a contour interval.
If the map meets NMAS standards, determine if the vertical and horizontal accuracies satisfy the ASPRS Class I Map Accuracy Standards (clearly showing all your steps) and discuss the main differences between the two standards.</td><td>7</td><td></td></tr>
</table>

6.	a) You are to perform Second Order Design (SOD) for horizontal positioning using the process of simulation. The relative positioning tolerance (limit on relative error ellipses at 95% confidence level) is to be 0.010 m and the potential observables will be angles and distances. Explain step-by-step and logically your whole plan on how to perform the simulation. Your plan must be complete enough for a computer programmer to use in developing a software application for the network design (do not be tempted to say, "Just use Pre-analysis or Design software"). You must provide the following for full marks: interpretation of the given tolerance, how to select potential geometry; input to the design and their underlying equations; design equations; general computational steps and appropriate equations and matrix expressions used at each step of the process (refer to the attached formula sheets for relevant equations and matrix expressions). Marks will be awarded depending on your demonstration of full understanding of the process involved and how much relevant details are logically provided.	20	
	b) The semi-major and semi-minor axis values of the standard relative error ellipse between two points P and Q (of length 1.5 km) of a horizontal survey network are 0.025 m and 0.015 m respectively. If this line PQ represents the least accurate section of the network, determine numerically if the network satisfies the Second Order accuracy standard of the Canadian horizontal control surveys, assuming the degrees of freedom for the least squares adjustment of the network is 12 with the a-priori variance factor of unit weight considered well-known.	4	
		100	

Some potentially useful formulae are given as follows:

$$v = \frac{Z_I + Z_{II} - 360}{2} \qquad \bar{z} = \frac{Z_I + (360 - Z_{II})}{2}$$

$$\frac{c}{\sin(z)} = \frac{Hz_I - (Hz_{II} - 180)}{2} \qquad \frac{t}{\tan(z)} + \frac{c}{\sin(z)} = \frac{Hz_I - (Hz_{II} - 180)}{2}$$

$$\textit{Corrected direction} = \textit{Measured direction} - \frac{(NR - NL) \times v''}{2 \tan z}$$

$$i_v = z - z' \text{ or } i_v = i \cos\alpha; \qquad i_T = Hz - Hz' \text{ or } i_T = \frac{i \sin\alpha}{\tan z}$$

Deformation: $\ell_2 - \ell_1 + V = Ad$; $d = \hat{x}_2 - \hat{x}_1$

$$F_c = \frac{\hat{d}^T Q_{\hat{d}}^{-1} \hat{d}}{\hat{\sigma}_0^2 u_d} < F(1 - \alpha_0, u_d, df_p); \qquad F_c = \frac{\hat{d}^T Q_{\hat{d}}^{-1} \hat{d}}{\hat{\sigma}_0^2 u_d} < \frac{\chi^2_{1-\alpha_0, df=u_d}}{u_d}$$

$\alpha = \frac{\delta\Delta h}{s}$ where $\delta\Delta h = \Delta h_{12t2} - \Delta h_{12\,t1}$.

$\sigma_\alpha = \frac{\sigma_{\delta\Delta h}}{s}$ where $\sigma_{\delta\Delta h} = \sqrt{\sigma^2_{\Delta h1} + \sigma^2_{\Delta h2}}$

EDM:

$$n_a = 1 + \frac{(n_g - 1)273.16p}{(273.16 + t)1013.25} \quad \text{(for } p \text{ in mb and } t \text{ in °C)}$$

$$N = (n-1) \times 10^6 \qquad \delta' = (N_{REF} - N_a)d' \times 10^{-6}$$

Standard pressure: 760 mmHg or 1013.25 mb; 0°C or 273.15 K

$$\hat{C} = \frac{M - (m_1 + m_2 + m_3 + m_4 + \ldots + m_n)}{n - 1}$$

Statistics:

$$|\Delta| = \sigma_\Delta \sqrt{\chi^2_{1-\alpha,df}} \qquad |\Delta| \leq z_{1-\alpha/2}\sigma_\Delta \qquad |\Delta| \leq t_{df,1-\alpha/2}\sigma_\Delta \qquad \hat{\sigma} \leq \sqrt{\frac{\chi^2_{1-\alpha,df}(\sigma)}{df}}$$

$$y = d\hat{x}^T C_{\hat{x}}^{-1} d\hat{x} \qquad \chi^2_{\frac{\alpha}{2},df} \leq \frac{(df)s^2}{\sigma^2} \leq \chi^2_{1-\frac{\alpha}{2},df} \qquad F_{1-\frac{\alpha}{2},df_1,df_2} \leq \frac{s_{01}^2}{s_{02}^2} \leq F_{\frac{\alpha}{2},df_1,df_2}$$

$$y < \chi^2_{u,1-\alpha} \qquad a_{(1-\alpha)100\%} = a_{st}\sqrt{\chi^2_{1-\alpha,df}} \quad \text{or} \quad a_{(1-\alpha)100\%} = a_{st}\sqrt{2F_{1-\alpha,df\ ,df2}}$$

Error propagation:

$$\sigma_{dp} = \frac{\sigma_p}{\sqrt{2n}} \qquad \sigma_{dp} = \frac{60}{M} \qquad \sigma_{\theta P} = \frac{\sigma_P}{\sqrt{n}} \qquad \sigma_{dr} = \frac{\sigma_r}{\sqrt{2n}} \qquad \sigma_{dr} = 2.5 \text{ div} \qquad \sigma_{\theta r} = \frac{\sigma_r}{\sqrt{n}}$$

$$\sigma_L = \sigma_v \cot z, \qquad \sigma_v = 0.2v'' \qquad \sigma_r = 2.5d'' \qquad \sigma_{\theta L} = \sigma_v\sqrt{\cot^2(Z_b) + \cot^2(Z_f)}$$

$$\sigma_i = \frac{(206265'')\sigma_{c3}}{S_1} \qquad \sigma_t = \frac{(206265'')\sigma_{c1}}{S_1} \qquad \sigma_{dc} = \frac{206265}{S}\sqrt{\sigma_{c3}^2 + \sigma_1^2}$$

$$\sigma_c = \pm 0.5mm\text{/m} \times \text{HI (m)} \qquad \sigma_c = \pm 0.1 \text{ mm} \qquad \sigma_c = \pm 0.1 \text{ mm/m} \times \text{HI (m)}$$

$$\sigma_{\theta i} = (206265'')\sigma_{c3}\sqrt{\left[\frac{S_1^2 + S_2^2 - 2S_1S_2\cos\theta}{S_1^2 S_2^2}\right]}$$

$$\sigma_{\theta i+t} = (206265'')\sqrt{\frac{\sigma_{c1}^2}{S_1^2} + \frac{\sigma_{c2}^2}{S_2^2} + \frac{\sigma_{c3}^2}{S_1^2 S_2^2}\left[S_1^2 + S_2^2 - 2S_1S_2\cos\theta\right]}$$

$$\sigma_P = \frac{45}{206265 \times M}S; \qquad \sigma_L = \left(\frac{\sigma_v}{206265}\right)S; \qquad \sigma_r = \frac{\ell}{2}\left(\frac{v_r}{206265}\right)^2$$

$$\sigma_d = \frac{S}{2R}\sigma_{k_h} \qquad \sigma_{ref} = \frac{S}{2R}\sigma_{k_V}$$

$$\ell = f(x) \qquad C_{\hat{x}} = \sigma_0^2\left(A^T PA\right)^{-1} \qquad \text{P} = Q^{-1}$$

$$s_{\Delta x}^2 = s_{x_1}^2 + s_{x_2}^2 - 2s_{x_1x_2} \qquad s_{\Delta x\Delta y} = s_{x_1y_1} + s_{x_2y_2} - s_{x_1y_2} - s_{y_1x_2} \qquad s_{\Delta y}^2 = s_{y_1}^2 + s_{y_2}^2 - 2s_{y_1y_2}$$

$$\lambda_1 = \frac{1}{2}\left(s_{\Delta x}^2 + s_{\Delta y}^2 + R\right) \qquad \lambda_2 = \frac{1}{2}\left(s_{\Delta x}^2 + s_{\Delta y}^2 - R\right) \qquad R = \left[\left(s_{\Delta x}^2 - s_{\Delta y}^2\right)^2 + 4s_{\Delta x\Delta y}^2\right]^{1/2}$$

$$a_s = \sqrt{\lambda_1} \qquad b_s = \sqrt{\lambda_2} \qquad a_{95} = k_{95}a_s \qquad b_{95} = k_{95}b_s$$

$$k_{95} = \sqrt{\chi^2_{2,1-0.05}} \qquad \beta = \arctan\left(\frac{s_{\Delta x\Delta y}}{\lambda_1 - s_{\Delta x}^2}\right)$$

Map projection and Reductions:

Meridian convergence: $\gamma = \dfrac{d\tan\phi\left(1 - e^2\sin^2\phi\right)^{1/2}}{a}$ a = 6378137 m; e = 0.081819191 or $\gamma = L\left(1+\frac{L^2}{3}\left(1+3\eta^2\right)\cos^2\phi\right)\sin\phi$

where $\eta^2 = e'^2\cos^2\phi$; $e'^2 = 0.006739496780$; $L = \left(\lambda - \lambda_0\right)$ (in radians); λ_0 is the longitude of the central meridian; and ϕ is the latitude of the given point.

$\alpha = A - \eta\tan\phi$ where $-\eta\tan\phi$ is Laplace correction

Horizontal Control Survey:

a = C(d +0.2) cm [where d is distance in km; C = 2 (First Order); C = 5 (Second Order)]

Vertical Control survey:

$\pm 3mm\sqrt{L}$ $\pm 4mm\sqrt{L}$ $\pm 8mm\sqrt{L}$ $\pm 24mm\sqrt{L}$ $\pm 120mm\sqrt{L}$

Map Accuracy Standards:

$Accuracy_x = SE \times \sqrt{\chi^2_{df,1-\alpha}}$ $Accuracy_y = SE \times \sqrt{\chi^2_{df,1-\alpha}}$

$Accuracy_z = SE \times z_{1-\alpha/2}$ $CMAS = SE \times z_{1-\alpha/2}$

$VMAS = CI/2$ $VMAS = SE \times z_{1-\alpha/2}$ $SE = RMSE$

Table 1: Normal Distribution table (upper tail area):

α	0.001	0.002	0.003	0.004	0.005	0.01	0.025	0.05	0.10
z_α	3.09	2.88	2.75	2.65	2.58	2.33	1.96	1.64	1.28

Table 2: Chi-Square Distribution table (lower tail area)

α	0.025	0.05	0.10	0.90	0.95	0.975	0.99	0.995
Degrees of freedom								
1	0.001	0.004	0.016	2.705	3.841	5.024	6.635	7.879
2	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
11	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	5.009	5.892	7.041	19.811	22.362	24.736	27.688	29.819
14	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
28	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993

Table 3: Table for Student-t distribution (α is upper tail area)

	t_α			
Degree of freedom	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36
6	1.49	1.94	2.45	3.14
11	1.363	1.796	2.201	2.718
12	1.356	1.782	2.179	2.681
13	1.350	1.771	2.160	2.650
14	1.345	1.761	2.145	2.624
15	1.341	1.753	2.131	2.602