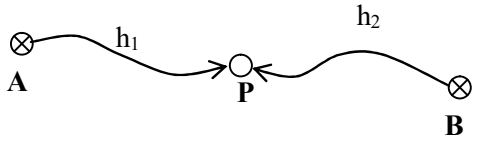
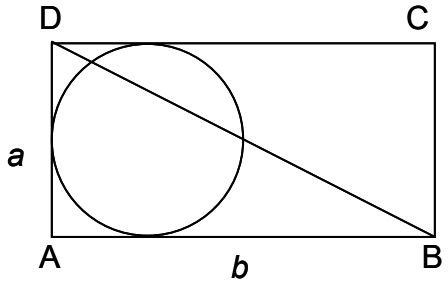
CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

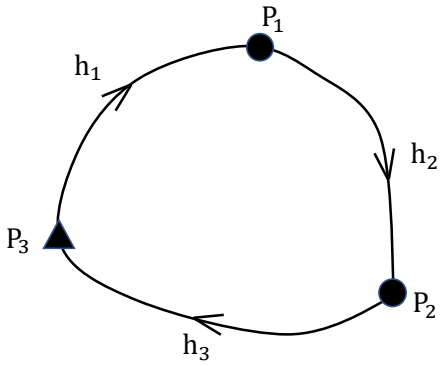
C2 - LEAST SQUARES & DATA ANALYSIS

March 2023

Note: This examination consists of 9 questions on 3 pages.

Time: 3 hours

Q. No		Marks Value	Earned
1.	Define and explain the following: a) Difference between precision and accuracy b) Difference between root mean square error and standard deviation c) Difference between covariance and correlation coefficient d) Internal and external reliability e) Type I and type II errors in statistical testing	15	
2.	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $C_x = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix} \text{ m}^2$	10	
3.	Given a leveling network below where A and B are known points, h_1 and h_2 are two height difference measurements with standard deviation of σ_1 and σ_2, respectively and $\sigma_1 = 2\ \sigma_2$. Determine the value of σ_1 and σ_2 so that the standard deviation of the height solution at P using least squares adjustment is equal to 2cm. 	10	
4.	Sides *a* and *b* are measured once each as follows: $\boldsymbol{l} = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix} \text{m}$ $\boldsymbol{C}_l = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \text{cm}^2$  a) Estimate the areas of triangle ABD and the circle shown inside the rectangle. b) Estimate the standard deviations of the quantities computed in Part (a). c) Estimate the correlation between the triangle and the circle estimates. d) Discuss the nature of the correlations computed in Part (c).	15	

5.	Given a leveling network with 60 observed height differences and 30 unknown points, use mathematical equations to explain which method of adjustment (parametric or conditional) you will recommend for this problem.	5	
6.	Consider that the shape of an object is defined by the following equation: $z_i = ax_i^3 + b\sin(y_i)$, i = 1, 2, 3. where z_i, x_i, y_i are observations with standard deviations σ_{z_i}, σ_{x_i}, σ_{y_i}, and a and b are parameters to be estimated. Derive the linearized form of this non-linear model for least squares adjustment including the required matrices and vectors.	10	
7.	Given a leveling network (see figure below) with three height difference observations (see table below). Assume that all observations were made with the same accuracy (σ = 1 mm). P_3 is a control point with known elevation 2.000 m. Conduct a conditional least squares adjustment on the leveling network. You are required to compute the following quantities: a) the adjusted observation residuals b) the variance-covariance matrix of the adjusted observation residuals c) the adjusted observations d) the variance-covariance matrix of the adjusted observations e) the a-posteriori variance factor  **Measurements** h_1: 2.125 m h_2: -1.343 m h_3: -0.779 m	15	
8.	Conduct a parametric least squares adjustment to the same data given in Problem 7. You are required to compute the following quantities: a) the adjusted elevations b) the variance-covariance matrix of the adjusted elevations c) the adjusted observations d) the observation residuals	10	

9. An angle has been measured independently 5 times with the same precision and the observed values are given in the following table. Test at the 95% level of confidence if the sample mean is significantly different from the true angle value 45°00′00″. (10)

α_1	α_2	α_3	α_4	α_5
45°00′05″	45°00′10″	44°59′58″	45°00′07″	44°59′54″

The critical value that might be required in the testing is provided in the following table:

	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

Total Marks: 100