CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-1 MATHEMATICS **October 2023**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 10 questions on 2 pages. **Marks**

Time: 3 hours

Q. No		Value	Earned
1.	Find the area of the region enclosed by the parabola $y = x^2$ and the curve $y = 2^x$. You may encounter an equation that you cannot solve using algebra. Use a numeric method instead.	10	
2.	(a) Use complex arithmetic to provide the following two expressions in rectangular form: $\frac{0.25}{3-\sqrt{-1}} \qquad \det\left(\begin{bmatrix} i-1 & 2i+1 \\ i & -i+3 \end{bmatrix}\right)$ (b) Convert the two complex numbers in (a) to polar form. Hint: rectangular form means z=a+bi; for the polar form you must provide a radius and an angle.	10	
3.	Find $\lim_{x \to 0} \frac{x - \sin x}{\tan x - x}$	10	
4.	(a) Complete the square to find the centre and radius of the circle $x^2 + 6x + y^2 + 8y + 1 = 0$. (b) Determine semimajor and semiminor axes of the ellipse $16x^2 + 3y^2 = 48$ Hint: for a circle, semimajor and semiminor axes a, b equal the radius r.	10	
5.	Find the second-order Taylor polynomial for $f(x) = \tan x$ about $a = \frac{\pi}{4}$. That is, find A, B, C in $\tan x \approx A + B\left(x - \frac{\pi}{4}\right) + C\left(x - \frac{\pi}{4}\right)^2$ where x is near $a = \frac{\pi}{4}$.	10	
6.	Use the integral test to test for convergence: $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$ Hint: consider the substitution $u = \ln x$ (which means $x = e^u$) and integration by parts.	10	

7.	(a) A theorem in calculus says that for well-behaved functions, $$\frac{\partial^2 z}{\partial x\,\partial y} = \frac{\partial^2 z}{\partial y\,\partial x}$$ Verify this theorem for $z = e^{-3x}\cos y$. (b) Find the dimensions that minimize the total cost of the material needed to construct the rectangular box which is open-topped with a volume of $600\, in^3$. The material for its bottom costs 6 cents per square inch, and the material for its four sides costs 5 cents per square inch.	10	
8.	Find the particular solution of the separable differential equation $$\frac{dy}{dx} + y\cos x = 6\cos x$$ satisfying the initial condition y(0)=8.	10	
9.	The currents running through an electrical system are given by the following system of equations. The three currents I_1, I_2, I_3 are measured in amps. Use Cramer's rule to find I_2. $$\begin{array}{rcrcrcl} I_1 & + & 2I_2 & - & I_3 & = & 0.425 \\ 3I_1 & - & I_2 & + & 2I_3 & = & 2.225 \\ 5I_1 & + & I_2 & + & 2I_3 & = & 3.775 \end{array}$$	10	
10.	(a) Calculate $\nabla \cdot \boldsymbol{v}$ if $$\boldsymbol{v} = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0\right)$$ Hint: $\nabla \cdot \boldsymbol{v}$ is the divergence of a vector field, which is defined as $$\nabla \cdot \boldsymbol{v} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$ for $\boldsymbol{v}(x,y,z) = (P(x,y,z), Q(x,y,z), R(x,y,z))$. (b) Find the divergence and the curl of $$\boldsymbol{v}(x,y,z) = (2x-y, 3y-2z, 7z-3x)$$ Hint: the curl of the differentiable vector field $\boldsymbol{v} = P\boldsymbol{i} + Q\boldsymbol{j} + R\boldsymbol{k}$ is the following vector field: $$\operatorname{curl} \boldsymbol{F} = \nabla \times \boldsymbol{F} = \det \begin{bmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{bmatrix}$$	10	
	Total Marks:	100	