

CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-1 MATHEMATICS

March 2023

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

EXAMINATION RESULTS FOR CANDIDATE NO.: 2303-011

Note: This examination consists of 10 questions on 3 pages.

Marks

Q. No

Time: 3 hours

Value Earned

1.	<p>(a) Manually (show all steps!) find eigenvalues and eigenvectors of the matrix</p> $A = \begin{bmatrix} 8 & -3 \\ 8 & -2 \end{bmatrix}$ <p>(b) Find $a, b, c, d, \lambda_1, \lambda_2, w, x, y, z$ such that</p>	10	
2.	<p>(a) If the position x of a body on a line (x-axis) at time t is</p> $x = \sqrt{3t^2 + 4}$ <p>find the velocity (first derivative) and acceleration (second derivative) at $t = 2$.</p> <p>(b) Which linear function in x and t best approximates the function</p>	10	
3.	<p>(a) If the result of the operations in part (a) or (b) does not exist, indicate it as DNE (does not exist). Manually (show all steps) find the determinants of</p> $B = \begin{bmatrix} -1 & 0 & -1 \\ 2 & -4 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} -1 & -3 \\ -4 & 4 \\ -1 & 2 \end{bmatrix}$	10	

4.	<p>(a) What is the distance along a great circle between Accra (5° and Pattaya ($12^\circ 56' N, 100^\circ 53'$)? Solve this problem using spherical trigonometry. Assume 6371 km for the Earth's radius. You may want to use the law of spherical triangles,</p> $\cos a = \cos b \cos c + \sin b \sin c \cos A$ <p>(b) Solve the problem again by finding the Cartesian coordinates of Accra and Pattaya assuming that the equator lies on the xy-plane of the coordinate system and the centre of the Earth (North Pole) lies on the z-axis). Then use the dot product to find the angle you find the great circle arc length. If ϕ is the polar angle, θ is the azimuthal angle, and ρ is the radius, then the spherical coordinates are:</p>	10	
5.	<p>(a) The gradient of a function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ is defined to be</p> $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$ <p>Find it for the function</p> $f(x, y, z) = \sin x - \cos y + e^z + \ln(xyz)$ <p>(b) Find the Laplacian for the function in part (a). The Laplacian is the sum of nonmixed second derivatives, as in</p>	10	
6.	<p>(a) Find the three cube roots of $2i - 2$, where $i^2 = -1$. It is not necessary to provide them in polar or rectangular coordinates.</p> <p>(b) Manually (show all steps!) determine a and b in</p> $\sqrt[3]{3 + 2i} = a + bi$	10	
7.	<p>(a) Use multiplication by the conjugate in order to find</p> $\frac{d}{dx} f(x) = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ <p>for $f(x) = \sqrt{3x+7}$.</p>	10	

8.	Marina had \$24,500 to invest. She divided the money into three accounts. At the end of the year, she had made \$1,300 in annual yield on each of the three accounts was 4%, 5.5%, and 7%. The amount of money in the 4% account was four times the amount in the 5.5% account.	10	
9.	The closest “solar system” to our home solar system is a cluster of stars, Rigil Kentaurus, Toliman, and Proxima Centauri. They are 4.2, 12.4, and 4.24 light years away from us. Proxima b is an Earth-sized planet in the habitable zone around Proxima Centauri. Breakthrough Starshot is a project led by Yuri Milner, Stephen Hawking, and Mark Zuckerberg to send a tiny interstellar probe to Proxima b. Imagine the probe travels a_n light years in the n -th decade such that $\sum_{n=0}^{\infty} a_n$ is a geometric series. Find the maximum distance that the probe can travel (can it reach Proxima b?). If the series diverges, you can indicate that the maximum distance is ∞ . Reminder: the formula for the limit of a geometric series is $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ for $ r < 1$.	10	
10.	<p>(a) Find $\frac{\vec{u} \cdot \vec{w}}{\ \vec{u}\ \ \vec{w}\ }$ for</p> $\vec{u} = \begin{pmatrix} 3 \\ -5 \\ -3 \end{pmatrix} \quad \vec{w} = \begin{pmatrix} -4 \\ -4 \\ 3 \end{pmatrix}$ <p>In an English sentence (as a geometric concept, not in numerical terms) describe the result of taking the arccosine of this number?</p>	10	
Total Marks:		100	