



Canadian Board of Examiners  
for Professional Surveyors

Conseil canadien des examinateurs  
pour les arpenteurs-géomètres

## S1 – Mathematics and Science

### Content

- This document is a high-level curriculum design which captures the key principles, competencies, learning outcomes and syllabus items proposed for the updated curriculum specific to S1 – Mathematics and Science.

**S1: MATHEMATICS AND SCIENCE**

**LEARNING OBJECTIVES**

- Establish the foundational knowledge in math, physics and computing required to be a surveyor
- Support subsequent learning

Key Principles	Motivation	Syllabus Items	Competencies and Learning Outcomes
<p><b>PROBABILITY AND STATISTICS</b></p>	<p>Surveyors must be able to deal with errors and uncertainty</p>	<ul style="list-style-type: none"> <li>• Probability theory: Samples, sample spaces, counting (Combinations, permutations) conditional probability, independence of events</li> <li>• Random variables and discrete and continuous distributions: binomial distribution, normal distribution,</li> <li>• Probability density functions and cumulative distribution functions,</li> <li>• Expected values, covariance and correlation</li> <li>• Distribution of the sample mean</li> <li>• Distribution of a linear combination, covariance propagation</li> <li>• Error Sources</li> <li>• Hypothesis testing</li> </ul>	<p><b>Competencies</b></p> <ul style="list-style-type: none"> <li>• Understand the ideas of random variables, independent and dependent samples, and related probabilities</li> <li>• Visualize random variables as probability distributions and related probability distributions to mean values and confidence regions</li> <li>• Know when to use which distributions and which tests</li> <li>• Recognize the existence of Type I and Type II errors in hypothesis testing</li> <li>• Recognize the probability density function plots for distributions such as normal, Student’s t, chi-squared, and Fisher</li> </ul> <p><b>Learning Outcomes</b></p> <ul style="list-style-type: none"> <li>• Choose appropriate error models and corresponding probability distributions for different types of surveying observations</li> <li>• Compute statistics and visualize data</li> <li>• Generate frequency, relative frequency (or probability), and probability density histograms</li> <li>• Apply a cumulative distribution function in order to compute probability values for given critical values of a specified distribution</li> <li>• Apply an inverse cumulative distribution function in order to extract critical values for given probability values of a specified distribution</li> <li>• Compute central tendency measures such as median, mode, and mean of a sample</li> <li>• Compute spread measures such as range, percentiles, variance, and standard deviation</li> </ul>

			<ul style="list-style-type: none"><li>• Convert an arbitrary normal random variance with specific mean and standard deviation to standardized normal distribution</li><li>• Differentiate between confidence and significance levels</li><li>• Compute covariances and correlation coefficients for pairs of multivariate quantities</li><li>• Compute confidence intervals for the mean, for the population variance, and for the ratio of two population variances</li><li>• Differentiate between systematic, random, and gross errors in observations</li><li>• Identify the source of errors (e.g., the environment, the data collection instrument, and/or the operator)</li><li>• Conduct a test of hypothesis for the population mean, for the population variance, and the ratio of two population variances</li></ul>
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<p><b>NUMERICAL METHODS</b></p>	<p>Surveyors must deal with numerical data because not all problems can be solved analytically</p>	<ul style="list-style-type: none"> <li>• Fundamentals of numerical methods; visualize, explore and then model</li> <li>• Numerical errors</li> <li>• Systems of linear equations</li> <li>• Interpolation and extrapolation</li> <li>• Numerical integration</li> <li>• Numerical aspects of solving systems of equations (least squares and the effects of geometry)</li> </ul>	<p><b>Competencies</b></p> <ul style="list-style-type: none"> <li>• Know why you choose which method by understanding the impact of numerical errors</li> <li>• Know how your choices can be affected by numeric layers</li> </ul> <p><b>Learning Outcomes</b></p> <ul style="list-style-type: none"> <li>• Correctly apply appropriate numerical methods to solve problems and meet required precision</li> <li>• Apply the rules of significant figures to determine the number of significant figures in a number</li> <li>• Demonstrate appropriate representation of data and results</li> <li>• Differentiate between analytical and numerical solutions to a problem</li> <li>• Convert between binary and decimal number representations and vice versa</li> <li>• Describe typical storage of floating point numbers in a computer system (e.g., single and double precision)</li> <li>• Explain terms such as overflow, underflow, machine precision, and round-off, chopping and truncation errors</li> <li>• Approximate order of magnitude for truncation error in Taylor series expansion</li> <li>• Mitigate unavoidable numerical errors in finite precision arithmetic especially for catastrophic/subtractive cancellation scenarios</li> </ul>

			<ul style="list-style-type: none"> <li>● Estimate absolute and relative (percent) error with respect to a reference value for various purposes including setting convergence criteria in iterative algorithms</li> <li>● Set up and solve a 'small' system of linear equations via elimination, decomposition/factorization and/or matrix inversion methods</li> <li>● Check a system of linear equations for rank deficiency and numerical ill-conditioning</li> <li>● Fit a data set to a simple curve such as a line, a quadratic or a cubic via linear least squares regression</li> <li>● Differentiate between interpolation and extrapolation</li> <li>● Perform interpolation on 'small' data sets using various methods (e.g., nearest neighbour interpolation or interpolating polynomials)</li> <li>● Perform interpolation on 'large' data sets using linear, quadratic and/or cubic splines in order to avoid Runge's phenomenon</li> <li>● Apply numerical differentiation for computing velocity and acceleration from a series of given positions</li> <li>● Apply numerical integration methods for computing areas and volumes</li> <li>● Plot results including appropriate title, axis labels, and units (graphical representation of numerical solutions)</li> <li>● Tabulate results including appropriate significant figures and units</li> <li>● Solve a 'large' system of linear equations via already existing computational libraries</li> </ul>
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<p><b>CALCULUS</b></p>	<p>Used to solve real-world problems</p> <p>Required to get the results that subsequently get solved with linear algebra</p>	<ul style="list-style-type: none"> <li>• Functions, continuity &amp; limits</li> <li>• Differentiation &amp; applications</li> <li>• Integration, quadratures &amp; applications</li> <li>• Plane curves, tangency &amp; curvature</li> <li>• Sequences, series &amp; Taylor expansions</li> <li>• Partial differentiation &amp; differential operators</li> <li>• Multiple integrals &amp; numerical approximations</li> <li>• Homogeneous ordinary differential equations</li> </ul>	<p><b>Competencies</b></p> <ul style="list-style-type: none"> <li>• Understand the importance, impact and application of using calculus to solve real-world problems</li> <li>• Appreciate the concept of small and continuous changes in inputs leading to small and continuous changes in outputs and how this relates to partial derivatives in both variance propagation equations and in least-squares estimation.</li> <li>• Understand the relationship between systems of first order ordinary differential equations and the state-space model used estimation.</li> </ul> <p><b>Learning Outcomes</b></p> <ul style="list-style-type: none"> <li>• Use calculus results to solve other problems (i.e. error propagation)</li> <li>• Define &amp; describe mathematical functions</li> <li>• Define &amp; illustrate continuity of a function at one point</li> <li>• Define &amp; evaluate mathematical limits</li> <li>• Define differentiability of a function at one point</li> <li>• Differentiate simple functions</li> <li>• Interpret the derivatives of a function</li> <li>• Define &amp; describe integration of a function</li> <li>• Integrate simple functions</li> <li>• Describe indefinite &amp; definite integrals</li> <li>• Evaluate numerically definite integrals</li> <li>• Formulate representations of plane curves</li> <li>• Describe the tangent to a curve at one point</li> </ul>

			<ul style="list-style-type: none"><li>• Describe the curvature of a curve at one point</li><li>• Describe sequences and series</li><li>• Define convergence of sequences and series</li><li>• Formulate tests of convergence for sequences and series</li><li>• Perform Taylor series expansions of simple functions</li><li>• Define and describe partial differentiation</li><li>• Partially differentiate simple functions</li><li>• Define gradient and Laplacian operations and describe their applications</li><li>• Define and describe multiple indefinite and definite integrals</li><li>• Describe numerical approximation techniques for multiple integrals</li><li>• Describe ordinary differential equations and their use in describing position and velocity as a function of time.</li></ul>
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<p><b>MATRICES &amp; LINEAR ALGEBRA</b></p>	<p>Surveyors estimate features with positions using observations; linear algebra is the tool used for estimating multiple unknowns</p>	<p><b>Vector operations &amp; analytical geometry</b></p> <ul style="list-style-type: none"> <li>● Real and complex vectors</li> <li>● Scalar and vector products of vectors</li> <li>● Analytical geometry equations and formulae</li> </ul> <p><b>Matrix algebra, linear equations &amp; transformations</b></p> <ul style="list-style-type: none"> <li>● Matrices and simple matrix algebra</li> <li>● Matrix representation of linear algebraic equations and solutions</li> <li>● Matrix representation of linear transformations</li> </ul> <p><b>Complex variables, linear spaces &amp; subspaces</b></p> <ul style="list-style-type: none"> <li>● Complex variables</li> <li>● Linear real and complex spaces and subspaces</li> <li>● Projections in real and complex spaces</li> </ul> <p><b>Quadratic forms, orthogonal &amp; unitary matrices</b></p> <ul style="list-style-type: none"> <li>● Quadratic forms and applications</li> <li>● Orthogonal and unitary matrices and their applications</li> </ul> <p><b>Eigen vectors &amp; values</b></p>	<p><b>Competencies</b></p> <ul style="list-style-type: none"> <li>● Understand the geometric interpretation of linear algebra</li> </ul> <p><b>Learning Outcomes</b></p> <ul style="list-style-type: none"> <li>● Populate vectors and matrices to perform certain vector/matrix operations</li> <li>● Perform matrix partition</li> <li>● Compute the trace of a square matrix</li> <li>● Solve systems of equations</li> <li>● Differentiate between row and column vectors</li> <li>● Compute the magnitude of a vector</li> <li>● Normalize a vector</li> <li>● Perform vector operations such as scaling, addition, dot product, and cross product</li> <li>● Find an angle between two vectors using their dot product</li> <li>● Project a vector onto another</li> <li>● Compute the eigenvalues and eigenvectors of a square matrix</li> <li>● Complete a vector triad using the cross product of two orthogonal vectors</li> <li>● Identify matrix dimensions</li> <li>● Perform matrix operations such as scaling, addition, multiplication, transposition</li> <li>● Compute the determinant of 2D and 3D matrices</li> <li>● Invert 2D and 3D matrices by hand and/or with a calculator</li> </ul>



			<ul style="list-style-type: none"><li>● Recognize special types of matrices such as rectangular, square, symmetric, diagonal, block diagonal, identity, etc.</li><li>● Convert a system of equations into vector and matrix form and vice versa</li><li>● Detect whether a system is rank deficient (i.e., singular)</li><li>● Compute the norm and condition number of a matrix</li><li>● Perform a transformation of a vector or matrix of coordinates from one system to another using a scaling factor, translation vector, and a rotation matrix</li><li>● Differentiate vector and/or matrix expressions w.r.t. a vector with parameters of interest using the numerator convention (also known as the Jacobian notation)</li></ul>
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<p><b>PHYSICS</b></p>	<p>Surveyors use measuring technologies that rely on certain physics principles</p>	<ul style="list-style-type: none"> <li>● Units of measure                             <ul style="list-style-type: none"> <li>○ SI system of units of measure</li> </ul> </li> <li>● Vectors and Scalars</li> <li>● Kinematics and dynamics</li> <li>● Optics (i.e. geometric and physical)</li> <li>● Electricity and Magnetism Waves</li> <li>● Energy, Acoustics and Wave Motion</li> <li>● Physics of the Earth</li> </ul>	<p><b>Competencies</b></p> <ul style="list-style-type: none"> <li>● Understand the laws of physics and how they enable and affect measuring technologies</li> <li>● Understand the Système International (SI) system of units of measure in quantity representations for; area, distance (length), frequency, volume, mass, time, electric current, force, energy, gravity and temperature;</li> <li>● Know the terms: mass, time, distance, force, acceleration, momentum, gravity, pressure;</li> <li>● Understand Newton’s Laws of motion</li> <li>● Know electric current terms; such as, ampere, charge, potential difference, and resistance;</li> <li>● Understand and use the laws of reflection and refraction;</li> <li>● Know the speed of sound, and the speed of light;</li> <li>● Know the terms: humidity, absolute humidity, relative humidity, saturation vapour pressure, equilibrium state</li> <li>● Know the atmospheric layers: ionosphere, mesosphere, stratosphere, troposphere, thermosphere, from earth’s surface</li> <li>● Understand Kepler's Laws of planetary motion (universal gravitation, gravitational field, orbital dynamics);</li> <li>● Understand spheroidal oscillations in elastic waves and seismic rays at earth’s surface;</li> <li>● Understand physics of the atmosphere and their effect on electromagnetic waves and measurements;</li> </ul>

			<p><b>Learning Outcomes</b></p> <ul style="list-style-type: none"> <li>● Carry out calculations symbolically and numerically</li> <li>● Use unit analysis to verify results/validity of equations</li> <li>● Describe physical quantity of anything that may be defined and measured;</li> <li>● Describe vectors quantities and magnitude and scalars quantities in magnitude</li> <li>● Perform vector addition graphically and mathematically and describe vector positions in coordinate systems</li> <li>● Perform linear motion calculations;</li> <li>● Perform speed calculation of objects (example; speed of a satellite orbiting the earth);</li> <li>● Perform calculations in: Work, Power, Uniform Circular Motion, Gravitational motion, and simple machines;</li> <li>● Describe Snell's Law;</li> <li>● Describe the conditions under which total internal reflection occurs;</li> <li>● Define different types of optical lens and find the focal length and power of a lens;</li> <li>● The use of Ray Diagrams and lens equation to find the position of objects and images;</li> <li>● Calculate magnification and angular magnification;</li> <li>● Describe the telescope as an optical instrument;</li> <li>● Describe circular wave patterns in sound wave;</li> <li>● Describe Moire pattern in sound propagation;</li> <li>● Describe sound intensity;</li> <li>● Explain sound wave equation variables.</li> <li>● Describe Gauss's Law and Coulomb's Law;</li> <li>● Calculate the energy and power supplied to electrical components;</li> <li>● Calculate magnetic flux density adjacent to wires carrying currents;</li> <li>● Describe Faraday's Law and Lenz's Law;</li> <li>● Describe the use of Maxwell's Equations;</li> </ul>
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			<ul style="list-style-type: none"><li>● Calculate the force on a moving charged particle;</li><li>● Calculate the force between parallel wires carrying a current;</li><li>● Calculate the value of the permeability of free space from the definition of the ampere;</li><li>● Calculate the torque on a rectangular coil carrying a current in a magnetic field;</li><li>● Describe light and sound as waves, and their wave motion;</li><li>● Describe stationary wave and know how stationary waves are formed;</li><li>● Describe wave harmonics;</li><li>● Explain the terms interference and diffraction of waves (doppler effect, resonance);</li><li>● Explain oscillatory of waves (periodic motion, extension);</li><li>● Describe the principle of superposition of waves;</li><li>● Explain the difference between Amplitude Modulation (AM), Frequency Modulation (FM), and Phase Modulation (PM);</li><li>● Explain energy sources and radiation principles;</li><li>● Describe the electromagnetic spectrum;</li><li>● Describe energy interactions with the earth's surface and the atmosphere;</li><li>● Explain spectral reflectance and response patterns from natural objects</li><li>● Use the Gauss coefficients in determining geomagnetic field;</li><li>● Express gravity as gradient of earth's geopotential;</li><li>● Calculate the velocity of electromagnetic energy as a rate of energy propagates through the atmosphere.</li></ul>
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<b>COMPUTING</b>	Surveyors are dealing with increasingly large data sets and need to be able to use software to process and manage data	<ul style="list-style-type: none"> <li>• Data types</li> <li>• Logic</li> <li>• Algorithms</li> <li>• Functions</li> <li>• Input/output</li> <li>• Simple program design</li> </ul>	<p><b>Competencies</b></p> <ul style="list-style-type: none"> <li>• Understand the methods used in commercial software packages (COTS)</li> <li>• Be able to translate problems to computer solvable programs</li> <li>• Develop strategies to manage data at all stages of Land Surveying work from acquisition to inclusion in final products, to long-term archiving. This may include preprocessing, quality assurance, converting, formatting for use with different COTS software, submission with Land Surveying products.</li> </ul> <p><b>Learning Outcomes</b></p> <ul style="list-style-type: none"> <li>• Understand the procedures involved in developing algorithms</li> <li>• Translate surveying problems into computer language</li> <li>• Understand the basics of functions, arrays and records</li> <li>• Solve problems using computer programming</li> <li>• Write simple functions to read and write ASCII and binary data from and to files appropriately formatted files</li> </ul>