CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS October 2022

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 5 questions on 3 pages. **Marks**

Time: 3 hours

Q. No		Value	Earned
1.	a) A 3TM zone (with False Easting of 304,800 m and the scale factor of central meridian of 0.99990) and a UTM zone have the same central meridian. Calculate the UTM coordinates of the point whose 3TM map coordinates are X = 274,800.000 m, Y = 5,500,000.000 m.	5	
	b) Determine the longitude coordinates (along the Equator) of the points where the scale factor distortion is minimal in UTM Zone 10N projection. What is that scale factor distortion?	5	
	c) In a large-scale cadastral mapping of a region (with 360 km East-West extent), a scaling accuracy ratio of 1/10,000 is required and a modified Transverse Mercator (MTM) projection is to be used. Determine (showing your computational steps) the scale factor (to 6 decimal places) and the number of projection zones for the region so that the scaling accuracy ratio remains within 1/10,000. The radius of the earth in the region can be taken as 6,371 km.	6	
2.	a) Discuss (with typical examples) the relationship between normal sections on a surface and the principal directions on that surface.	3	
	b) Given the general scale factors along the meridians and the parallels for a map projection system as $1/\cos\phi$ and the Gaussian surface element as $f = 0$ (where ϕ is the latitude), answer the following: i. Determine mathematically if the map projection is conformal or equivalent and show that both distortion properties cannot be satisfied at the same time (showing all of your mathematical expressions and substitutions of values into them for full marks).	5	
	ii. Determine the generally values of the semi-major and semi-minor axes of the Tissot indicatrices for the map projection and clearly describe their practical application and interpretation in a mapping based on this projection.	4	
3.	a) Right Ascension system is an inertial reference system. Explain why it is an inertial system, and discuss how its curvilinear coordinates are measured (including their units and zero points of the coordinates).	5	
	b) Explain one important difference between the International Terrestrial Reference system and the Instantaneous terrestrial system.	2	
	c) Describe the origin and coordinate axes of the coordinate system that is commonly used for satellite positioning at each specific instant of time the satellite is observed and explain four of the important parameters needed to convert coordinates in this coordinate system into geocentric coordinate system.	8	
	d) Describe the parameters of 14-parameter transformation, and explain the purpose of the transformation.	4	
	e) Explain the important differences between UT1 and UTC (stating their full names and the main properties distinguishing them).	4	

4.	On an UTM projection, point A is located on the eastern secant line while point B is located on a line where the scale factor is 1.0001 towards the eastern boundary. If the UTM plane bearing of line AB is 45º30′10.5", determine the grid bearing of the line (assuming the Gaussian mean radius of the earth in the region is 6,382,129.599 m).	20	
5.	a) Using well labelled sketches only, illustrate the Transverse Mercator (TM) and the Mercator projections in the Northern hemisphere; give one sketch for the TM projection and the other sketch for the Mercator projection. The sketches must show the projections of the Equator, Central Meridian, parallels and meridians with appropriate relationship between lines of the graticule clearly illustrated. b) What are the ellipsoidal (latitude and longitude) coordinates of the points where the grid convergence values are minimal and maximal in a TM projection (Northern hemisphere) having a zone width of 6° and a Central Meridian at 123°W? Calculate the meridian convergence values corresponding to those points (assuming the Northern boundary of the zone is at 84°N). c) Discuss the properties of TM and the Mercator projections with regard to aspects, distortion characteristics, developable surfaces, specific applications, and edge-matching possibilities.	16 6 7	
		100	

Some potentially useful formulae are given as follows:

$$T - t = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - x_0$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i.

UTM average line scale factor, $\bar{k}_{UTM} = k_0\left[1 + \frac{x_u^2}{6R_m^2}\left(1 + \frac{x_u^2}{36R_m^2}\right)\right]$;

where $x_i = x_i^{UTM} - x_0$; $x_u^2 = x_1^2 + x_1x_2 + x_2^2$

UTM point scale factor, $k_{UTM} = k_0\left[1 + \frac{\Delta x^2}{2R_m^2}\right]$, where $\Delta x = x^{UTM} - x_0$

$$k_{UTM} = k_0\left[1 + \frac{L^2}{2}\cos^2\phi\right]$$

k_0 is scale factor of Central Meridian and x_0 is the False easting value (or 500 000 m) $L = (\lambda - \lambda_0)$ (in radians) for a given longitude λ; and λ_0 is the longitude of the central meridian.

Grid convergence, $\gamma = L\left(1 + \frac{L^2}{3}(1 + 3\eta^2)\cos^2\phi\right)\sin\phi$

where $\eta^2 = e'^2\cos^2\phi$; $e'^2 = 0.006739496780$; $L = (\lambda - \lambda_0)$ (in radians); λ_0 is the longitude of the central meridian; and ϕ is the latitude of the given point.

Geodetic bearing: $\alpha = t + \gamma + (T - t)$

$$Sf = \frac{R_m}{R_m + H_m}$$

Transformation Formulas:

$$X_{(target)} = k_{0(target)} X_G + X_{0(target)}$$
$$Y_{(target)} = k_{0(target)} Y_G$$
$$X_G = \frac{\left[X_{(original)} - X_{0(original)}\right]}{k_{0(original)}}$$
$$Y_G = \frac{Y_{(original)}}{k_{0(original)}}$$

ITRF:

$$\mathbf{r}(t) = \mathbf{r}_0 + \dot{\mathbf{r}}\,(t - t_0)$$

where $\mathbf{r}_0$ and $\dot{\mathbf{r}}$ are the position and velocity respectively at $\mathbf{t}_0$.

Distortion Formulas:

Given: $X = f(\phi, \lambda)$ $\quad Y = g(\phi, \lambda)$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2};\; m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2 \cos^2\phi};\; p = \frac{2(f_\phi f_\lambda + g_\phi g_\lambda)}{R^2 \cos\phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A_p' \;;$$

$$\sin A_p' = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{\left(f_\lambda g_\phi - f_\phi g_\lambda\right)^2 + \left(f_\phi f_\lambda + g_\phi g_\lambda\right)^2}}$$

Equivalency Condition: $f_\lambda g_\phi - f_\phi g_\lambda = \pm R^2 \cos\phi$

Cauchy-Riemann Equations: $f_\phi = -\frac{1}{\cos\phi} g_\lambda$; $\quad g_\phi = \frac{1}{\cos\phi} f_\lambda$