CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-3 ADVANCED SURVEYING **October 2022**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 7 questions on 6 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	a) In order to provide confidence in the area determination of an irregular quadrilateral lot ABCD in a flat topography, the surveyor has designed to measure the horizontal angles in the lot to a standard deviation of ±3″ and the distances to a standard deviation of ±0.002 m. The critical section of the lot is point C of the lot where the surveyor will measure lengths CB and CA, and angle BCA (θ_1). For pre-analysis, the approximate measurements made from a large-scale topographic map of the lot are CB = 72 m; CA = 125 m and θ_1 =33˚. With consideration for pointing, reading, centering and leveling errors, determine numerically if Leica TC2003 with angle-measurement standard deviation (ISO 17123-2) as 0.5″ and distance-measurement standard deviation (ISO 17123-4) as 1 mm + 1 ppm will achieve the designed standard deviations of angle and distance measurements (suggesting the number of sets of measurements required). Assume the instrument will be set up on a tripod with height above the setup point as 1.65 m and the centering will be done with laser plummet; and the centering error of each target will be 1 mm.	14	
	b) The semi-major and semi-minor axis values of the standard relative error ellipse between two points P and Q (of length 1.5 km) of a horizontal survey network are 0.025 m and 0.015 m respectively. If this line PQ represents the least accurate section of the network, determine numerically if the network satisfies the Second Order accuracy standard of the Canadian horizontal control surveys, assuming the degrees of freedom for the least squares adjustment of the network is 12 with the a-priori variance factor of unit weight considered well-known.	4	

2.	Leica DNA03 digital level (with appropriate invar rods) with stated accuracy of 0.3 mm/km double-run (according to ISO17123-2 standard) was calibrated according to ISO standard17123-2 procedure. The data collected during the procedure were adjusted by least squares method with the adjusted values (with the number of degrees of freedom as 14) as: $\widehat{dz_0}$ = -0.52 mm ±0.25 mm; $\hat{\sigma}_{dh}$ = ±0.18 mm where $\widehat{dz_0}$ is the estimated difference of the zero-point offsets of the invar rods used and its standard deviation, and $\hat{\sigma}_{dh}$ is the experimental standard deviation of elevation difference. Answer the following: a) Determine at 95% confidence level if $\widehat{dz_0}$ is significantly different from zero (providing the hypotheses, tests and conclusions). b) Determine statistically if the performance of the digital level is still acceptable at 95% confidence level (clearly providing null and alternative hypotheses, statistic, test and conclusion; and including appropriate formulae for error propagation, the error propagation equations and logical assumptions involved with the explanation of any symbol involved). c) Explain with justification six important Quality Assurance (QA) /Quality Control (QC) measures that you would follow during the level calibration process.	3 8 6	
3.	a) A local plane coordinate system was established at the collar of a shaft at latitude of 59°30′N (with grid convergence equal to zero). At a depth of 2km, a tunnel runs approximately in a westerly direction from the shaft. An open traverse follows the tunnel with stations along one side. Gyro-azimuths, based on a well-calibrated Gyromat 3000 automated gyro station, were measured at regular intervals in order to “control” the orientation of the tunnel. The specified accuracy of an azimuth determination with the equipment and procedures is ± 3″ and the average deflection of the vertical along the tunnel is -5". Apart from the internal corrections that are automatically done by the gyro station to the gyro measurements, explain two other important systematic corrections, with justification and suggestions of their numerical values (with calculation steps), that should be applied to the azimuths, observed at 4 km (westerly) from the shaft in order to convert it to a grid azimuth in the surface coordinate system. b) Clearly explain two major corrections usually applied to tape measurements in elevation transfer into an underground mine that are not normally applied when a tape is used to determine horizontal distances.	7 2	

4.	a) In a deformation survey, the datum-independent displacement vector ($\hat{d}$) and the corresponding cofactor matrix ($Q_{\hat{d}}$) for a monitored point as follows: $\hat{d} = \begin{bmatrix} dE \\ dN \end{bmatrix} = \begin{bmatrix} -0.013m \\ -0.006m \end{bmatrix}$ $Q_{\hat{\hat{d}}} = \begin{bmatrix} 2.0323E-5 & 1.0665E-6 \\ 1.0665E-6 & 1.2796E-5 \end{bmatrix} m^2$ with the pooled variance factor as 0.518, the number of degrees of freedom for the pooled variance as 28 and the number of network points being considered for deformation analysis as 5. Determine the statistical significance of the displacement at this point at the significance level, $\alpha = 5\%$, assuming the a priori variance factor of unit weight is well known, and explain how you would have checked if the measurements in the two epochs involved are compatible before the deformation analysis. b) Discuss how surveyor can use geodetic level as a geotechnical tiltmeter (explaining the observables, formulae involved, and one possible advantage over tiltmeter).	8 6	
5.	You are required to carry out a topographic survey of a proposed construction project site of about 5 ha. The expected drawing scale is 1:1000; ground elevations are to be shot at 15 m grid spacing and the final drawings must be delivered with a 0.5 m contour interval and must meet the NMAS Standards. Some of the specifications for the typical map standards, such as NMAS and ASPRS are as follows: • For NMAS, the horizontal tolerance is 0.82 mm for map scales larger than 1:20,000 and the elevation should be accurate to within one-half a contour interval. • For ASPRS, the maximum allowable error (limiting RMSE) for X or Y coordinates of well-defined points for map of 1:1000 is 0.25 m; and the elevation should be accurate to within one-third a contour interval. If the map meets NMAS standards, determine if the vertical and horizontal accuracies satisfy the ASPRS Class I Map Accuracy Standards (clearly showing all your steps) and discuss two main differences between the two standards.	7	
6.	a) Precise point positioning (PPP) is becoming an attractive alternative to real-time kinematic (RTK) in GNSS surveying. Discuss four of the important benefits of PPP technique compared to double-difference RTK technique. b) GNSS system validation is one of the important GNSS field procedures that may be required prior to a GNSS control survey. Discuss briefly the key elements of the GNSS measurement validation procedure, purpose (including quantities to be validated), how often it is done, and explain a typical statistical test to check the external accuracy of the validation (including necessary statistical formulae with symbols well defined, and the expected degrees of freedom and confidence level).	8 13	

7.	a) Describe the Zero-order Design (ZOD), First-order Design (FOD) and Second-order Design (SOD) problems of deformation monitoring network. b) The allowable vertical breakthrough tolerance between two breakthrough points A and B in a tunneling survey is 0.010 m. The design of vertical control for the tunnelling survey is to follow the usual procedure of carrying out independent design of surface and underground vertical control networks. In the simulation of the underground network design based on minimal constraint least squares procedure, the standard deviation of the elevation difference between points A and B is estimated as 0.005 m; similarly, for the surface network design, the covariance matrix for the two breakthrough points (in the order A and B) is given as $$C_{AB} = \begin{bmatrix} 2.895E-6 & 1.298E-6 \\ 1.298E-6 & 4.165E-6 \end{bmatrix} \text{m}^2$$ c) Interpret the meaning of the tolerance specified for the tunneling survey, use this interpretation to determine if the allowable vertical breakthrough tolerance is satisfied by the current design, and explain one practical approach of minimizing the effects of refraction during the tunneling survey.	5 9	
		100	

Some potentially useful formulae are given as follows:

$$v = \frac{Z_I + Z_{II} - 360}{2} \qquad \bar{z} = \frac{Z_I + (360 - Z_{II})}{2}$$

$$\frac{c}{\sin(z)} = \frac{Hz_I - (Hz_{II} - 180)}{2} \qquad \frac{t}{\tan(z)} + \frac{c}{\sin(z)} = \frac{Hz_I - (Hz_{II} - 180)}{2}$$

$$\textit{Corrected direction} = \textit{Measured direction} - \frac{(NR - NL) \times v''}{2 \tan z}$$

$$i_v = z - z' \text{ or } i_v = i \cos\alpha; \qquad i_T = Hz - Hz' \text{ or } i_T = \frac{i \sin\alpha}{\tan z}$$

Deformation: $\ell_2 - \ell_1 + V = Ad$; $\qquad d = \hat{x}_2 - \hat{x}_1$

$$F_c = \frac{\hat{d}^T Q_{\hat{d}}^{-1} \hat{d}}{\hat{\sigma}_0^2 u_d} < F(1 - \alpha_0, u_d, df_p); \qquad F_c = \frac{\hat{d}^T Q_{\hat{d}}^{-1} \hat{d}}{\hat{\sigma}_0^2 u_d} < \frac{\chi^2_{1-\alpha_0, df=u_d}}{u_d}$$

$$\alpha = \frac{\delta\Delta h}{s}$$ where $\delta\Delta h = \Delta h_{12t2} - \Delta h_{12t1}$.

$$\sigma_\alpha = \frac{\sigma_{\delta\Delta h}}{s}$$ where $\sigma_{\delta\Delta h} = \sqrt{\sigma^2_{\Delta h1} + \sigma^2_{\Delta h2}}$

EDM:

$$n_a = 1 + \frac{(n_g - 1)273.16p}{(273.16 + t)1013.25}$$ (for p in mb and t in °C)

$$N = (n-1) \times 10^6 \qquad \delta' = (N_{REF} - N_a)d' \times 10^{-6}$$

Standard pressure: 760 mmHg or 1013.25 mb; 0°C or 273.15 K

$$\hat{C} = \frac{M - (m_1 + m_2 + m_3 + m_4 + \ldots + m_n)}{n-1}$$

Statistics:

$|\Delta| = \sigma_\Delta \sqrt{\chi^2_{1-\alpha,df}}$ $\quad$ $|\Delta| \le z_{1-\alpha/2}\sigma_\Delta$ $\quad$ $|\Delta| \le t_{df,1-\alpha/2}\sigma_\Delta$ $\quad$ $\hat{\sigma} \le \sqrt{\frac{\chi^2_{1-\alpha,df}(\sigma)}{df}}$

$y = d\hat{x}^T C_{\hat{x}}^{-1} d\hat{x}$ $\quad$ $\chi^2_{\frac{\alpha}{2},df} \le \frac{(df)s^2}{\sigma^2} \le \chi^2_{1-\frac{\alpha}{2},df}$ $\quad$ $F_{1-\frac{\alpha}{2},df_1,df_2} \le \frac{s_{01}^2}{s_{02}^2} \le F_{\frac{\alpha}{2},df_1,df_2}$

$y < \chi^2_{u,1-\alpha}$ $\quad$ $a_{(1-\alpha)100\%} = a_{st}\sqrt{\chi^2_{1-\alpha,df}}$ or $a_{(1-\alpha)100\%} = a_{st}\sqrt{2F_{1-\alpha,df1,df2}}$

Error propagation:

$\sigma_{dp} = \frac{\sigma_p}{\sqrt{2n}}$ $\quad$ $\sigma_{dp} = \frac{60}{M}$ $\quad$ $\sigma_{\theta P} = \frac{\sigma_P}{\sqrt{n}}$ $\quad$ $\sigma_{dr} = \frac{\sigma_r}{\sqrt{2n}}$ $\quad$ $\sigma_{dr} = 2.5$ div $\quad$ $\sigma_{\theta r} = \frac{\sigma_r}{\sqrt{n}}$

$\sigma_L = \sigma_v \cot z$, $\quad$ $\sigma_v = 0.2v''$ $\quad$ $\sigma_r = 2.5d''$ $\quad$ $\sigma_{\theta L} = \sigma_v\sqrt{\cot^2(Z_b) + \cot^2(Z_f)}$

$\sigma_i = \frac{(206265'')\sigma_{c3}}{S_1}$ $\quad$ $\sigma_t = \frac{(206265'')\sigma_{c1}}{S_1}$ $\quad$ $\sigma_{dc} = \frac{206265}{S}\sqrt{\sigma_{c3}^2 + \sigma_{c1}^2}$

$\sigma_c = \pm 0.5mm$/m × HI (m) $\quad$ $\sigma_c = \pm 0.1$ mm $\quad$ $\sigma_c = \pm 0.1$ mm/m × HI (m)

$$\sigma_{\theta i} = (206265'')\sigma_{c3}\sqrt{\left[\frac{S_1^2 + S_2^2 - 2S_1S_2\cos\theta}{S_1^2S_2^2}\right]}$$

$\sigma_P = \frac{45}{206265 \times M}S$; $\quad$ $\sigma_L = \left(\frac{\sigma_v}{206265}\right)S$; $\quad$ $\sigma_r = \frac{\ell}{2}\left(\frac{v_r}{206265}\right)^2$

$\sigma_d = \frac{S}{2R}\sigma_{k_h}$ $\quad$ $\sigma_{ref} = \frac{S}{2R}\sigma_{k_V}$

$\ell = f(x)$ $\quad$ $C_{\hat{x}} = \sigma_0^2\left(A^T PA\right)^{-1}$ $\quad$ $P = Q^{-1}$

$s_{\Delta x}^2 = s_{x_1}^2 + s_{x_2}^2 - 2s_{x_1x_2}$ $\quad$ $s_{\Delta x\Delta y} = s_{x_1y_1} + s_{x_2y_2} - s_{x_1y_2} - s_{y_1x_2}$ $\quad$ $s_{\Delta y}^2 = s_{y_1}^2 + s_{y_2}^2 - 2s_{y_1y_2}$

$\lambda_1 = \frac{1}{2}\left(s_{\Delta x}^2 + s_{\Delta y}^2 + R\right)$ $\quad$ $\lambda_2 = \frac{1}{2}\left(s_{\Delta x}^2 + s_{\Delta y}^2 - R\right)$ $\quad$ $R = \left[\left(s_{\Delta x}^2 - s_{\Delta y}^2\right)^2 + 4s_{\Delta x\Delta y}^2\right]^{1/2}$

$a_s = \sqrt{\lambda_1}$ $\quad$ $b_s = \sqrt{\lambda_2}$ $\quad$ $a_{95} = k_{95}a_s$ $\quad$ $b_{95} = k_{95}b_s$

$k_{95} = \sqrt{\chi^2_{2\,,1-0.05}}$ $\quad$ $\beta = \arctan\left(\frac{s_{\Delta x\Delta y}}{\lambda_1 - s_{\Delta x}^2}\right)$

Map projection and Reductions:

Meridian convergence: $\gamma = \frac{d\tan\phi\left(1 - e^2\sin^2\phi\right)^{1/2}}{a}$ $\quad$ a = 6378137 m; e = 0.081819191

or $\gamma = L\left(1 + \frac{L^2}{3}\left(1 + 3\eta^2\right)\cos^2\phi\right)\sin\phi$

where $\eta^2 = e'^2\cos^2\phi$; e'^2 =0.006739496780; $L = \left(\lambda - \lambda_0\right)$ (in radians); λ_0 is the longitude of the central meridian; and ϕ is the latitude of the given point.

$\alpha = A - \eta\tan\phi$ $\quad$ where $-\eta\tan\phi$ is Laplace correction

Horizontal Control Survey:
a = C(d +0.2) cm [where d is distance in km; C = 2 (First Order); C = 5 (Second Order)]

Vertical Control survey:
$\pm 3mm\sqrt{L}$ $\pm 4mm\sqrt{L}$ $\pm 8mm\sqrt{L}$ $\pm 24mm\sqrt{L}$ $\pm 120mm\sqrt{L}$

Map Accuracy Standards:

$Accuracy_x = SE \times \sqrt{\chi^2_{df,1-\alpha}}$ $Accuracy_y = SE \times \sqrt{\chi^2_{df,1-\alpha}}$

$Accuracy_z = SE \times z_{1-\alpha/2}$ $CMAS = SE \times z_{1-\alpha/2}$

$VMAS = CI/2$ $VMAS = SE \times z_{1-\alpha/2}$ $SE = RMSE$

Table 1: Normal Distribution table (upper tail area):

α	0.001	0.002	0.003	0.004	0.005	0.01	0.025	0.05	0.10
z_α	3.09	2.88	2.75	2.65	2.58	2.33	1.96	1.64	1.28

Table 2: Chi-Square Distribution table (lower tail area)

α	0.025	0.05	0.10	0.90	0.95	0.975	0.99	0.995
Degrees of freedom								
1	0.001	0.004	0.016	2.705	3.841	5.024	6.635	7.879
2	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
11	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	5.009	5.892	7.041	19.811	22.362	24.736	27.688	29.819
14	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
28	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993

Table 3: Table for Student-t distribution (α is upper tail area)

	t_α			
Degree of freedom	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36
6	1.49	1.94	2.45	3.14
11	1.363	1.796	2.201	2.718
12	1.356	1.782	2.179	2.681
13	1.350	1.771	2.160	2.650
14	1.345	1.761	2.145	2.624
15	1.341	1.753	2.131	2.602