CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C2 - LEAST SQUARES & DATA ANALYSIS   October 2022

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 10 questions on 3 pages.

Time: 3 hours

Q. No		Marks Value	Marks Earned
1.	Explain the differences between the following: a) Precision and accuracy b) Root mean square error and standard deviation c) Covariance and correlation coefficient d) Internal and external reliability e) Type I and type II errors in statistical testing	15	
2.	The distance between two points has been independently measured 10 times with the same precision $\sigma = 2$ cm. Determine the precision of the obtained mean distance.	5	
3.	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $$C_x = \begin{bmatrix} 0.0484 & 0.0246 \\ 0.0246 & 0.0196 \end{bmatrix} \text{ m}^2$$	10	
4.	Given the following mathematical model $f(l, x) = 0 \qquad C_l \quad C_x$ where f is the vector of mathematical models, x is the vector of unknown parameters and C_x is its variance matrix, l is the vector of observations and C_l is its variance matrix, a) Linearize the mathematical model b) Formulate the variation function c) Derive the least squares normal equation	15	
5.	Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$: $$C_\ell = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$ and the function $x = \ell_1 + \ell_2$, determine C_x.	5	

6. An angle has been measured independently 5 times with the same precision and the observed values are given in the following table. Test at the 95% level of confidence if the sample mean is significantly different from the true angle value 45°00’00”. (10)

α_1	α_2	α_3	α_4	α_5
45°00’05”	45°00’10”	44°59’58”	45°00’07”	44°59’54”

The critical value that might be required in the testing is provided in the following table:

	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

7. A distance has been independently measured 4 times and its sample unit variance obtained from the adjustment $\hat{\sigma}_0^2$ is equal to 1.44 cm. If the a-priori standard deviation σ_0 is 1.0 cm, conduct a statistic test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. The critical values that might be required in the testing are provided in the following table: (10)

α	0.001	0.01	0.025	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=3}$	16.26	11.34	9.35	7.82	6.25

where $\chi^2_{\alpha,\ \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=3}}^{\infty} \chi^2(x)dx$ and υ is the degree of freedom.

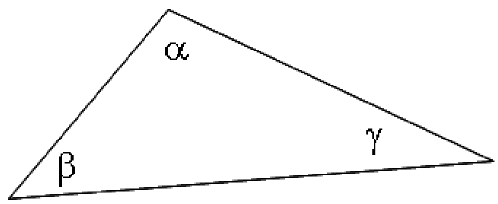
8. Given a leveling network with 200 observed height differences and 50 unknown points, use mathematical equations to explain which method of least squares adjustment (parametric or conditional) you will recommend for this problem. (5)

9.

Given the angle measurements of a triangle along with their standard deviations, conduct a conditional least squares adjustment. You are required to compute the following quantities:

a) the estimated residuals
b) the variance-covariance matrix of the estimated residuals
c) the estimated observations
d) the variance-covariance matrix of the estimated observations
e) the estimated variance factor

Angle	Measurement	Standard Deviation
α	104º38'56"	6.7"
β	43º17'35"	9.9"
γ	32º03'14"	4.3"



15

10.

Conduct a parametric least squares adjustment to the same data given in Problem 9. You are required to compute the following quantities:

a) the estimated parameters
b) the variance-covariance matrix of the estimated parameters
c) the estimated difference between α and β
d) the variance of the estimated difference between α and β

10

Total Marks: 100