CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-1 MATHEMATICS **October 2022**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 10 questions on 2 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	a) The displacement of a mass on a spring suspended from the ceiling is given by $$y(t) = 10e^{-\frac{t}{2}}\cos\left(\frac{\pi t}{8}\right)$$ Compute the velocity function $v(t) = y'(t)$. b) Find a time t_0 at which the mass reaches a high or low point of its oscillation.	5 5	
2.	Here is the definition of divergence for a vector-valued function $\boldsymbol{F} : (x, y, z) \to (P(x, y, z), Q(x, y, z), R(x, y, z))$. Note that P, Q, R are real-valued functions. $$\operatorname{div}\boldsymbol{F} = \nabla\cdot\boldsymbol{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$ Find the divergence (which is a real number) of $$\boldsymbol{F}(x, y, z) = (xe^{y})\boldsymbol{i} + (z\sin y)\boldsymbol{j} + (xy\ln z)\boldsymbol{k}$$ at $(-3, 0, 2)$, where $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are the standard unit vectors of three-dimensional space.	10	
3.	The curl of the differentiable vector field $\boldsymbol{F} = P\boldsymbol{i} + Q\boldsymbol{j} + R\boldsymbol{k}$ (think of $\boldsymbol{F}, P, Q, R$ as in the last question) is the following vector field: $$\operatorname{curl}\boldsymbol{F} = \nabla\times\boldsymbol{F} = \det\begin{bmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{bmatrix}$$ where $\frac{\partial}{\partial x}P$ is interpreted to be the partial derivative of P with respect to x (that's a bit of notational abuse). Find the curl (which is a vector) of $$\boldsymbol{F}(x, y, z) = (xe^{y})\boldsymbol{i} + (z\sin y)\boldsymbol{j} + (xy\ln z)\boldsymbol{k}$$ at $(3, \frac{\pi}{2}, e)$	10	
4.	Calculate trace and determinant of A, where $i^2 = -1$. $$A = \begin{bmatrix} 3 & 3+2i & 4+i \\ 3-2i & -2 & i \\ 4-i & -i & 1 \end{bmatrix}$$	10	

5.	a) Find the eigenvalues of $$B = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$$ and find one eigenvector for each eigenvalue. b) If λ_1, λ_2 are the eigenvalues and v_1, v_2 are the eigenvectors of B, then we can decompose $$B = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} v_1 & v_2 \end{bmatrix}^{-1}$$ Verify this decomposition by executing the matrix multiplication.	5 5	
6.	Solve the equation $4x^2 - 24x + 37 = 0$ in the complex numbers.	10	
7.	a) Chaitali and Amulya go to a concession stand to buy fruit. Chaitali buys 5 bananas and 3 apples and spends \$13.50. Amulya buys 1 banana and 5 apples and spends 20 cents more than Chaitali. How much do bananas and apples cost at the concession stand? Find the answer to this question using Cramer's Rule. b) Let $Ax = b$ be a system of linear equations, where A is an invertible matrix, b is a known vector, and x is an unknown vector. Then you can easily find x using linear algebra: $x = A^{-1}b$. Use this method to solve the problem in the last question (prices of bananas and apples).	5 5	
8.	Find $h'''(x)$ for $h(x) = \tan^2 x$. Note that $\frac{d}{dx} \tan x = \sec^2 x$ and $\frac{d}{dx} \sec x = \tan x \sec x$.	10	
9.	Consider the plane containing the three points $$\begin{aligned} P &= (1,3,0) \\ Q &= (3,4,-3) \\ R &= (3,6,2) \end{aligned}$$ Finding a plane equation is easy if we have a normal vector to the plane. The cross product $\vec{PQ} \times \vec{PR}$ is such a normal vector. Calculate this cross product.	10	
10.	In a right spherical triangle with $C = 90°$, one side $a = 126°5'20''$ and its opposing angle $A = 105°55'30''$. Calculate the side c.	10	
	Total Marks:	100	