CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS — March 2021

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 5 questions on 3 pages. **Marks**

Time: 3 hours

Q. No		Value	Earned
1.	Scientists prefer to work with geodetic coordinates (latitude, longitude) as reference system compared to map projection coordinates (northing, easting). Explain five important advantages and five important disadvantages of using geodetic coordinates as nation-wide grid reference system compared with map projection coordinates.	15	
2.	a) Explain how the easting and northing coordinates, meridian convergence and scale factor variations in Transverse Mercator projections mathematically relate to those in the Universal Transverse Mercator projections.	5	
	b) Given the Universal Transverse Mercator (UTM) map coordinates (in UTM Zone 10) of points B and C as follows:		
	Station B: Easting (m) 564,702.284; Northing (m) 5,588,965.983 Station C: Easting (m) 563,836.008; Northing (m) 5,580,487.376		
	The Gaussian mean radius of the earth in the region, R_m = 6,382,129.599 m. Calculate the geodetic azimuth (to the tenth of a second) and ellipsoidal distance (to 3 decimal places) of line B to C if the convergence of meridian at point B is 1°29′00.0″.	17	
	c) A 3TM zone (with False Easting of 304,800 m and the scale factor of central meridian of 0.99990) and a UTM zone have the same central meridian. Calculate the UTM coordinates of the point whose 3TM map coordinates are X = 274,800.000 m, Y = 5,500,000.000 m.	5	
3.	a) Maps that cover large areas of the globe usually have stated scale called the nominal scale. However, on such maps one simply cannot measure the distance from any point on a map to another, multiply by the nominal scale, and expect to have obtained a correct distance. Explain why and clearly discuss (providing formulae and a demonstration of understanding of the terms involved) how you would obtain a correct distance from such maps.	7	
	b) Geocentric (Cartesian) coordinate system is commonly used in modern geodetic positioning. Discuss (with justification) two important advantages and three disadvantages of using geocentric (Cartesian) coordinates to represent positions.	10	
	c) Discuss the concept of Tissot indicatrix and suggest how it can be used in practice.	5	
	d) You are required to transform three-dimensional baseline vectors determined in GNSS survey to map projection coordinates. Explain (without providing specific formulae) the steps for the transformation, indicating types of surfaces, types of coordinates and types of transformation functions involved in each of the steps.	6	

Station table for 2 b):

Station	Easting (m)	Northing (m)
B	564,702.284	5,588,965.983
C	563,836.008	5,580,487.376

4.	Answer all of the following: a) Name one inertial reference system (or an approximation) and compare and contrast its properties (origin, Cartesian coordinate axes orientations, primary plane, why it is inertial or not) with those of the International Terrestrial Reference System (ITRS). b) Discuss three classes of time scales that are each related to some natural observable phenomenon (stating the phenomena they are related to and how). c) Explain one important difference between the ITRS and the Instantaneous terrestrial (IT) system.	10 3 2	
5.	The scale factor (k) at any point (x, y) on a UTM projection can be determined using the following formula: $k = k_0\left[1+\frac{(x-x_0)^2}{2R^2}\right]$ where k_0 and x_0 are the scale factor and the false Easting coordinate at the central meridian, respectively, and R is the mean radius of the earth. In a large-scale cadastral mapping of a region (with 360 km East-West extent), a scaling accuracy ratio of 1/10,000 is required and a modified Transverse Mercator projection (similar to UTM) is to be used. The radius of the earth in the region can be taken as 6,371 km. a) Determine the number of zones (showing the computational steps followed) and the scale factor (to 6 decimal places) to be used at the central meridian so that the scaling accuracy ratio remains within 1/10,000. b) What is the distance between the two secant lines in a zone while still maintaining the scaling accuracy of 1/10,000 in the region? c) If a single zone is used for the whole mapping region, what would the worst scaling accuracy ratio for the zone be, assuming the scale factor determined in (a) is adopted for the central meridian?	6 5 4	
		100	

Some potentially useful formulae are given as follows:

$$\textit{Arc-to-chord correction} = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - x_0$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i.

UTM average line scale factor, $\bar{k}_{UTM} = k_0\left[1+\frac{x_u^2}{6R_m^2}\left(1+\frac{x_u^2}{36R_m^2}\right)\right]$;

where $x_i = x_i^{UTM} - x_0$; $x_u^2 = x_1^2 + x_1x_2 + x_2^2$

UTM point scale factor, $k_{UTM} = k_0\left[1+\frac{\Delta x^2}{2R_m^2}\right]$, where $\Delta x = x^{UTM} - x_0$

$$k_{UTM} = k_0\left[1+\frac{L^2}{2}\cos^2\phi\right]$$

k_0 is scale factor of Central Meridian and x_0 is the False easting value (or 500 000 m)
$L = (\lambda - \lambda_0)$ (in radians) for a given longitude λ; and λ_0 is the longitude of the central meridian.

Grid convergence, $\gamma = L\left(1+\frac{L^2}{3}\left(1+3\eta^2\right)\cos^2\phi\right)\sin\phi$

where $\eta^2 = e'^2\cos^2\phi$; e'^2 =0.006739496780; $L = \left(\lambda - \lambda_0\right)$ (in radians); λ_0 is the longitude of the central meridian; and ϕ is the latitude of the given point.

Geodetic bearing: $\alpha = t + \gamma + (T - t)$

$$Sf = \frac{R_m}{R_m + H_m}$$

Transformation Formulas:

$$X_{(target)} = k_{0(target)}X_G + X_{0(target)}$$
$$Y_{(target)} = k_{0(target)}Y_G$$
$$X_G = \frac{\left[X_{(original)} - X_{0(original)}\right]}{k_{0(original)}}$$
$$Y_G = \frac{Y_{(original)}}{k_{0(original)}}$$

ITRF:

$$\mathbf{r}(t) = \mathbf{r}_0 + \dot{\mathbf{r}}\,(t - t_0)$$

where $\mathbf{r}_0$ and $\dot{\mathbf{r}}$ are the position and velocity respectively at $\mathbf{t}_0$.

Distortion Formulas:

Given: $X = f\left(\phi,\lambda\right)$ $\qquad$ $Y = g\left(\phi,\lambda\right)$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2};\ m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2\cos^2\phi};\ p = \frac{2\left(f_\phi f_\lambda + g_\phi g_\lambda\right)}{R^2\cos\phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A_p' \ ;$$

$$\sin A_p' = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{\left(f_\lambda g_\phi - f_\phi g_\lambda\right)^2 + \left(f_\phi f_\lambda + g_\phi g_\lambda\right)^2}}$$

Equivalency Condition: $f_\lambda g_\phi - f_\phi g_\lambda = \pm R^2\cos\phi$

Cauchy-Riemann Equations: $f_\phi = -\frac{1}{\cos\phi}g_\lambda$; $\qquad g_\phi = \frac{1}{\cos\phi}f_\lambda$