CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-3 ADVANCED SURVEYING

March 2021

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 5 questions on 4 pages. **Marks**

Time: 3 hours

Q. No		Value	Earned
1.	a) Explain the important differences in the objectives of the First-order Design (FOD) and Second-order Design (SOD) of deformation monitoring network and geodetic positioning network.	8	
	b) You are to perform SOD for horizontal positioning using the process of simulation. The relative positioning tolerance (limit on relative error ellipses at 95% confidence level) is to be 0.010 m and the potential observables will be angles and distances. Explain step-by-step and logically your whole plan on how to perform the simulation. Your plan must be complete enough for a computer programmer to use in developing a software application for the network design (do not be tempted to say, "Just use Pre-analysis or Design software"). You must provide the following for full marks: interpretation of the given tolerance, how to select potential geometry; input to the design and their underlying equations; design equations; general computational steps and appropriate equations and matrix expressions used at each step of the process (refer to the attached formula sheets for relevant equations and matrix expressions). Marks will be awarded depending on your demonstration of full understanding of the process involved and how much relevant details are logically provided.	20	
	c) Explain six important precautions that you would take to ensure that the final design tolerance (after the simulation process) would be realized during the measurement scheme.	6	
2.	a) The Canadian First Order levelling specifications require that the discrepancy between independent forward and backward levelling runs (at 95% confidence) is not to exceed ±4 mm $\sqrt{L}$ (with L in kilometers). Derive an expression for the standard deviation of mean elevation difference of forward and backward levelling runs over a section of L km, and determine the numerical value for the standard deviation of an elevation difference of one-way levelling run over a section of 1 km (you must show all necessary equations and assumptions for full marks).	7	
	b) Using modern total stations [angular accuracy of ± 1″ and a distance accuracy of ± 1 mm ± 1 ppm according to ISO Standards] has the potential for competing with precise differential levelling. Assuming the total station is used in a leap-frog trigonometric levelling procedure with imposed balanced maximum sight length of 150 m, determine numerically (stating any assumptions made) whether the levelling result will satisfy the First Order specification. [Let the average zenith angle at each station to back sight target be 91.5° and to foresight target as 88.5°, and assume that the standard deviation of height difference measurement between back sight and foresight targets at each setup is zero.]	15	

3.	A leveling instrument that has not been used for over 20 years is to be used for a survey project. The manufacturer claims, following DIN 18723 (or ISO 17123, now), that the equipment has a standard deviation of ±0.2 mm over 1-km double-run leveling. Since there is no record of any testing or calibration of this particular instrument, explain (with reasons) all of the necessary setting out, six important Quality Assurance (QA) /Quality Control (QC) measures, and the field procedure (including the number and types of measurements made in the field) that you would recommend following in order to determine whether the level is capable of behaving as the manufacturer claimed. Explain four quantities that will be determined from the processing of the measurements and fully discuss the statistical tests that will be performed on some of the quantities in order to determine whether the level is capable of behaving as the manufacturer claimed.	20	
4.	Two survey crews A and B measured the length of a horizontal baseline using the same EODMI instrument (with precision ± 3 mm ± 2 ppm and the reference refractivity as 281.949). Crew A measured the whole baseline and obtained the overall length of the baseline (corrected wrongly for meteorological condition using refractivity of 300.000 instead of the correct value of 305.520) as 1799.921 m. Crew B measured the baseline in two equal sections (with each section measured independently) and obtained the meteorologically corrected overall length of the baseline as 1799.931 m. Answer the following, assuming each of the crews was able to center their instrument to an accuracy of 0.8 mm and their target to 0.8 mm. Determine if there is any significant difference (at 99% confidence level) between the two lengths obtained by crews A and B.	10	
5.	A closed-loop horizontal traverse of 4 points is to be run in a fairly flat and homogeneous terrain using Leica 802 total station with specified standard deviation of 2″ according to ISO 17123-3 Standard. Assume each traverse leg is approximately 100 m long; the included angle at each traverse point is approximately 90°; the targets and total station are to be centered on tripods using tribrachs with optical plummets; heights of targets and instrument are to be set at 1.6 m and the included angle at each traverse point is to be measured in one set. With consideration for leveling, centering, pointing and reading errors at each setup point, determine if the traverse will satisfy the allowable angular misclosure of 15″ at 95% confidence level.	14	
		100	

Some potentially useful formulae are given as follows:

$$v = \frac{Z_I + Z_{II} - 360}{2} \qquad \bar{z} = \frac{Z_I + (360 - Z_{II})}{2}$$

$$\frac{c}{\sin(z)} = \frac{Hz_I - (Hz_{II} - 180)}{2} \qquad \frac{t}{\tan(z)} + \frac{c}{\sin(z)} = \frac{Hz_I - (Hz_{II} - 180)}{2}$$

$$\textit{Corrected direction} = \textit{Measured direction} - \frac{(NR - NL) \times v''}{2 \tan z}$$

$$i_v = z - z' \text{ or } i_v = i \cos\alpha ; \qquad i_T = Hz - Hz' \text{ or } i_T = \frac{i \sin\alpha}{\tan z}$$

Deformation: $\ell_2 - \ell_1 + V = Ad$; $\qquad d = \hat{x}_2 - \hat{x}_1$

$$F_c = \frac{\hat{d}^T Q_{\hat{d}}^{-1} \hat{d}}{\hat{\sigma}_0^2 u_d} < F(\alpha_0, u_d, df_p); \qquad F_c = \frac{\hat{d}^T Q_{\hat{d}}^{-1} \hat{d}}{\hat{\sigma}_0^2 u_d} < \frac{\chi^2_{\alpha_0, df=u_d}}{u_d}$$

$\alpha = \frac{\delta\Delta h}{s}$ where $\delta\Delta h = \Delta h_{12t2} - \Delta_{h12\,t1}$.

$\sigma_\alpha = \frac{\sigma_{\delta\Delta h}}{s}$ where $\sigma_{\delta\Delta h} = \sqrt{\sigma^2_{\Delta h1} + \sigma^2_{\Delta h2}}$

EDM:

$$n_a = 1 + \frac{(n_g - 1)273.16p}{(273.16 + t)1013.25}$$ (for p in mb and t in °C)

$N = (n-1) \times 10^6 \qquad \delta' = (N_{REF} - N_a)d' \times 10^{-6}$

Standard pressure: 760 mmHg or 1013.25 mb; 0°C or 273.15 K

$$\hat{C} = \frac{M - (m_1 + m_2 + m_3 + m_4 + ... + m_n)}{n-1}$$

Levelling: $\pm 3mm\sqrt{L}$ $\quad \pm 4mm\sqrt{L}$ $\quad \pm 8mm\sqrt{L}$ $\quad \pm 24mm\sqrt{L}$ $\quad \pm 120mm\sqrt{L}$

Statistics:

$$|\Delta| = \sigma_\Delta \sqrt{\chi^2_{df,\alpha}} \qquad |\Delta| \le z_{\alpha/2}\sigma_\Delta \qquad |\Delta| \le t_{df,\alpha/2}\sigma_\Delta \qquad \hat{\sigma} \le \sqrt{\frac{\chi^2_{\alpha,df}(\sigma)}{df}}$$

$$\sigma_{dp} = \frac{\sigma_p}{\sqrt{2n}} \qquad \sigma_{dp} = \frac{60}{M} \qquad \sigma_{\theta P} = \frac{\sigma_P}{\sqrt{n}} \qquad \sigma_{dr} = \frac{\sigma_r}{\sqrt{2n}} \qquad \sigma_{dr} = 2.5 \text{ div} \qquad \sigma_{\theta r} = \frac{\sigma_r}{\sqrt{n}}$$

$$\sigma_L = \sigma_v \cot z, \qquad \sigma_v = 0.2v'' \qquad \sigma_r = 2.5d'' \qquad \sigma_{\theta L} = \sigma_v \sqrt{\cot^2(Z_b) + \cot^2(Z_f)}$$

$$\sigma_i = \frac{(206265'')\sigma_{c3}}{S_1} \qquad \sigma_t = \frac{(206265'')\sigma_{c1}}{S_1} \qquad \sigma_{dc} = \frac{206265}{S}\sqrt{\sigma^2_{c3} + \sigma^2_1}$$

$\sigma_c = \pm 0.5mm$ /m × HI (m) $\qquad \sigma_c = \pm 0.1$ mm $\qquad \sigma_c = \pm 0.1$ mm/m × HI (m)

$$\sigma_{\theta i} = (206265'')\sigma_{c3}\sqrt{\left[\frac{S_1^2 + S_2^2 - 2S_1S_2\cos\theta}{S_1^2 S_2^2}\right]}$$

$$\sigma_{\theta i+t} = (206265'')\sqrt{\frac{\sigma^2_{c1}}{S_1^2} + \frac{\sigma^2_{c2}}{S_2^2} + \frac{\sigma^2_{c3}}{S_1^2 S_2^2}\left[S_1^2 + S_2^2 - 2S_1S_2\cos\theta\right]}$$

$$\sigma_P = \frac{45}{206265 \times M}S; \qquad \sigma_L = \left(\frac{\sigma_v}{206265}\right)S; \qquad \sigma_r = \frac{\ell}{2}\left(\frac{v_r}{206265}\right)^2$$

$$\sigma_d = \frac{S}{2R}\sigma_{k_h} \qquad \sigma_{ref} = \frac{S}{2R}\sigma_{k_V}$$

$$\ell = f(x) \qquad C_{\hat{x}} = \sigma_0^2\left(A^T PA\right)^{-1} \qquad P = Q^{-1}$$

$$s^2_{\Delta x} = s^2_{x_1} + s^2_{x_2} - 2s_{x_1x_2} \qquad s_{\Delta x\Delta y} = s_{x_1y_1} + s_{x_2y_2} - s_{x_1y_2} - s_{y_1x_2} \qquad s^2_{\Delta y} = s^2_{y_1} + s^2_{y_2} - 2s_{y_1y_2}$$

$$\lambda_1 = \frac{1}{2}\left(s^2_{\Delta x} + s^2_{\Delta y} + R\right) \qquad \lambda_2 = \frac{1}{2}\left(s^2_{\Delta x} + s^2_{\Delta y} - R\right) \qquad R = \left[\left(s^2_{\Delta x} - s^2_{\Delta y}\right)^2 + 4s^2_{\Delta x\Delta y}\right]^{1/2}$$

$$a_s = \sqrt{\lambda_1} \qquad b_s = \sqrt{\lambda_2} \qquad a_{95} = k_{95}a_s \qquad b_{95} = k_{95}b_s$$

$$k_{95} = \sqrt{\chi^2_{2\ ,1-0.05}} \qquad \beta = \arctan\left(\frac{s_{\Delta x\Delta y}}{\lambda_1 - s^2_{\Delta x}}\right)$$

Table 1: Normal Distribution table (upper tail area):

α	0.001	0.002	0.003	0.004	0.005	0.01	0.025	0.05	0.10
z_α	3.09	2.88	2.75	2.65	2.58	2.33	1.96	1.64	1.28

Table 2: Chi-Square Distribution table (lower tail area)

α	0.025	0.05	0.10	0.90	0.95	0.975	0.99	0.995
Degrees of freedom								
1	0.001	0.004	0.016	2.705	3.841	5.024	6.635	7.879
2	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
11	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	5.009	5.892	7.041	19.811	22.362	24.736	27.688	29.819
14	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801

Table 3: Table for Student-t distribution (α is upper tail area)

	t_α			
Degree of freedom	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36
6	1.49	1.94	2.45	3.14
11	1.363	1.796	2.201	2.718
12	1.356	1.782	2.179	2.681
13	1.350	1.771	2.160	2.650
14	1.345	1.761	2.145	2.624
15	1.341	1.753	2.131	2.602