CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C2 - LEAST SQUARES ESTIMATION & DATA ANALYSIS March 2021

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 10 questions on 4 pages.

		Marks	
Q. No	Time: 3 hours	Value	Earned
1.	Define and explain the following: a) Difference between precision and accuracy b) Difference between root mean square error and standard deviation c) Difference between covariance and correlation coefficient d) Internal and external reliability e) Type I and type II errors in statistical testing	15	
2.	The distance between Point A and Point B has been independently measured 5 times with the same precision using a distance measuring device and the standard deviation of the obtained mean distance is 1.58 cm. Determine the precision of the distance measurement. 	5	
3.	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $$C_x = \begin{bmatrix} 0.0484 & 0.0246 \\ 0.0246 & 0.0196 \end{bmatrix} \text{ m}^2$$	10	
4.	Given the following mathematical model $f(l, x) = 0 \quad C_l \quad C_x$ where f is the vector of mathematical models, x is the vector of unknown parameters and C_x is its variance matrix, *l* is the vector of observations and C_l is its variance matrix: a) Linearize the mathematical model b) Formulate the variation function c) Derive the least squares normal equation	15	

5.

Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$:

$$C_\ell = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

and the function $x = \ell_1 + \ell_2$, determine C_x.

5

6.

An angle has been measured independently 5 times with the same precision and the observed values are given in the following table. Test at the 95% level of confidence if the sample mean is significantly different from the true angle value 45°00′00″.

α_1	α_2	α_3	α_4	α_5
45°00′05″	45°00′10″	44°59′58″	45°00′07″	44°59′54″

The critical value that might be required in the testing is provided in the following table:

	t_α			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

10

7.

A distance has been independently measured 4 times and its sample unit variance obtained from the adjustment $\hat{\sigma}_0^2$ is equal to 1.44 cm. If the a-priori standard deviation σ_0 is 1.0 cm, conduct a statistic test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. The critical values that might be required in the testing are provided in the following table:

α	0.001	0.01	0.025	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=3}$	16.26	11.34	9.35	7.82	6.25

where $\chi^2_{\alpha,\ \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=3}}^{\infty} \chi^2(x)dx$ and υ is the degree of freedom.

10

8.

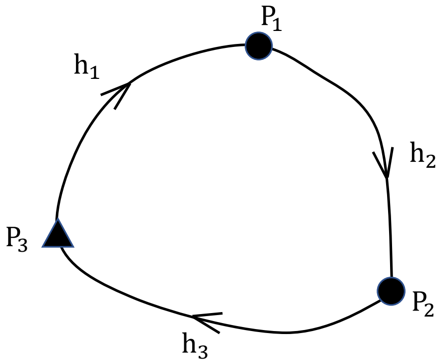
Given a leveling network with 60 observed height differences and 30 unknown points, use mathematical equations to explain which method of adjustment (parametric or conditional) you will recommend to be used for this problem.

5

9.

Given a leveling network (see figure below) with three height difference observations (see table below). Assume that all observations were made with the same accuracy (σ = 1 mm). P_3 is a control point with known elevation 2.000 m. Conduct a conditional least squares adjustment on the leveling network. You are required to compute the following quantities:

a) the adjusted observation residuals
b) the variance-covariance matrix of the adjusted observation residuals
c) the adjusted observations
d) the variance-covariance matrix of the adjusted observations
e) the a-posteriori variance factor



	Measurements
h_1	2.125 m
h_2	-1.343 m
h_3	-0.779 m

15

10.	Conduct a parametric least squares adjustment to the same data given in Problem 9. You are required to compute the following quantities: a) the adjusted elevations b) the variance-covariance matrix of the adjusted elevations c) the adjusted observations d) the observation residuals	10	
	Total Marks:	100	