C-1 MATHEMATICS

October 2020

Marks

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 10 questions on 2 pages.

Q. No	Time: 3 hours	<u>Value</u>	Earned
1.	a) Integrate $\int \frac{x}{x+7} dx$. b) Evaluate $\int_{1}^{2} x \sin(x^{2}) dx$.	5	
2.	Solve the isosceles spherical triangle ABC , where $a=c=79^{\circ}17'$ and $A=C=59^{\circ}37'$. In other words, find b and B .	10	
3.	a) Find the eigenvalues of $A_2 = \begin{bmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{bmatrix}$. b) Solve for x, y, z : $\begin{bmatrix} 3 & 4 & -2 \\ 1 & 2 & -1 \\ 3 & -1 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 9 \\ 6 \end{bmatrix}$	5	
4.	a) Find $\sum_{n=1}^{\infty} e^{-n}$ if it exists (five significant digits). b) Find $\sum_{n=1}^{\infty} \frac{6n+3}{n^2(n+1)^2}$. Hint: use partial fractions and telescoping series.	5	
	a) Consider a quadratic equation $a^2x^2 + bx - c$ with real numbers a, b, c		

and c > 0. Consider the discriminant and indicate how many solutions the

Find an orthonormal basis for the plane whose equation is 2x - 3y + 7z = 0.

b) What is the angle between the two vectors a and b in the previous ques-

a) What is the cross product $a \times b$ for $a = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$ and $b = \begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix}$?

b) What is the inverse B^{-1} of $B = \begin{bmatrix} -5 & 4 \\ -1 & 2 \end{bmatrix}$?

tion?

equation has.

5.

6.

7.

5

10

5

5

8.	a) Use the definition of the tangent function $\tan x$ and the quotient rule to show that $\frac{d}{dx} \tan x = \sec^2 x$.	5	
	b) Find the equation of the line tangent to the graph of $y = 2 \ln(x)$ at $x = 3$.	5	
	a) Find the Taylor polynomial of order 3 generated by $f(x) = \sin x$ at $a = \frac{\pi}{4}$.	5	
9.	b) Find the Mclaurin series for $f(x) = \cosh x$. Recall that $\cosh x = \frac{1}{2} (e^x + e^{-x})$.	5	
10	a) Let $f(x) = x + 4$ and $g(x) = 2x - 5$. Find $(f \circ g)(x)$ and its inverse.	5	
10.	b) Find the three cube roots of 27 in the complex numbers.	5	
	Total Marks:	100	