CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS **October 2020**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 5 questions on 3 pages. **Marks**

<table>
<tr><th>Q. No</th><th>Time: 3 hours</th><th>Value</th><th>Earned</th></tr>
<tr><td>1.</td><td>a) Sketch the graticule appearance of the Universal Transverse Mercator (UTM) projection for Zone 10 and label the following on the sketch: longitude of Central Meridian, Equator, False Northing and False Easting coordinates of origin, latitude limits, longitudes of zone boundaries, scale factor at the Central Meridian.</td><td>12</td><td></td></tr>
<tr><td></td><td>b) Explain how the easting and northing coordinates, meridian convergence and scale factor variations in Transverse Mercator projections mathematically relate to those in the Universal Transverse Mercator projections.</td><td>5</td><td></td></tr>
<tr><td></td><td>c) Given the Universal Transverse Mercator (UTM) map coordinates (in UTM Zone 10) of points B and C as follows:
<table>
<tr><th>Station</th><th>Easting (m)</th><th>Northing (m)</th></tr>
<tr><td>B</td><td>564,702.284</td><td>5,588,965.983</td></tr>
<tr><td>C</td><td>563,836.008</td><td>5,580,487.376</td></tr>
</table>
The Gaussian mean radius of the earth in the region, R_m = 6,382,129.599 m. Calculate the geodetic azimuth (to the tenth of a second) and ellipsoidal distance (to 3 decimal places) of line B to C if the convergence of meridian at point B is 1°29′00.0″.</td><td>17</td><td></td></tr>
<tr><td></td><td>d) A 3TM zone (with False Easting of 304,800 m and the scale factor of central meridian of 0.99990) and a UTM zone have the same central meridian. Calculate the UTM coordinates of the point whose 3TM map coordinates are X = 274,800.000 m, Y = 5,500,000.000 m.</td><td>5</td><td></td></tr>
<tr><td>2.</td><td>Answer all of the following:
a) Name one inertial reference system (or an approximation) and compare and contrast its properties (origin, Cartesian coordinate axes orientations, primary plane, why it is inertial) with those of the International Terrestrial Reference System (ITRS).</td><td>10</td><td></td></tr>
<tr><td></td><td>b) Explain one important difference between the ITRS and the Instantaneous Terrestrial (IT) system.</td><td>2</td><td></td></tr>
<tr><td></td><td>c) Explain how an orbital coordinate system is defined (describing the origin and coordinate axes) and describe three of the important parameters needed to convert coordinates in an orbital system to geocentric coordinate system.</td><td>7</td><td></td></tr>
<tr><td></td><td>d) Clearly explain the essential differences between CGVD28 and CGVD2013 and discuss their uses in geomatics (demonstrating in your explanation and discussion your understanding of the full name and meaning of each term).</td><td>7</td><td></td></tr>
<tr><td>3.</td><td>Answer all of the following.
a) Explain the important differences between UT1 and UTC (stating their full names and what distinguish them).</td><td>4</td><td></td></tr>
<tr><td></td><td>b) Discuss three classes of time scales that are each related to some natural observable phenomenon (stating the phenomena they are related to and how).</td><td>3</td><td></td></tr>
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4.	Answer all of the following. a) A typical cadastral survey plan usually shows, among other details, the horizontal ground-level distances and the corresponding astronomic bearings of boundaries of a given land parcel; these distances and bearings are different from the plotted corresponding distances and bearings on the plan. Explain the differences and provide the theoretical and practical procedures for making the quantities equivalent. b) Discuss briefly the concept of Tissot indicatrix and clearly describe its practical applications using an equivalent projection as an example. c) What is (T-t) correction? This correction is composed of two parts in conformal stereographic double projections; explain the two parts.	7 7 4	
5.	The map projection equations relating map projection coordinates (x, y) with the corresponding geographic coordinates (ϕ, λ) can be given as $x = R\lambda \qquad\qquad y = R\sin\phi$ where R is the mean radius of the spherical earth. Determine (showing your mathematical derivations) if the projection is equivalent using two different approaches (the conclusions of the two approaches must be identical).	10	
		100	

Some potentially useful formulae are given as follows:

$$\textit{Arc-to-chord correction} = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - x_0$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i.

UTM average line scale factor, $\bar{k}_{UTM} = k_0\left[1 + \frac{x_u^2}{6R_m^2}\left(1 + \frac{x_u^2}{36R_m^2}\right)\right]$;

where $x_i = x_i^{UTM} - x_0$; $\quad x_u^2 = x_1^2 + x_1x_2 + x_2^2$

UTM point scale factor, $k_{UTM} = k_0\left[1 + \frac{\Delta x^2}{2R_m^2}\right]$, where $\Delta x = x^{UTM} - x_0$

$$k_{UTM} = k_0\left[1 + \frac{L^2}{2}\cos^2\phi\right]$$

k_0 is scale factor of Central Meridian and x_0 is the False easting value (or 500 000 m) $L = (\lambda - \lambda_0)$ (in radians) for a given longitude λ; and λ_0 is the longitude of the central meridian.

Grid convergence, $\gamma = L\left(1 + \frac{L^2}{3}(1 + 3\eta^2)\cos^2\phi\right)\sin\phi$

where $\eta^2 = e'^2\cos^2\phi$; $e'^2 = 0.006739496780$; $L = (\lambda - \lambda_0)$ (in radians); λ_0 is the longitude of the central meridian; and ϕ is the latitude of the given point.

Geodetic bearing: $\alpha = t + \gamma + (T - t)$

$$Sf = \frac{R_m}{R_m + H_m}$$

Transformation Formulas:

$$X_{(target)} = k_{0(target)} X_G + X_{0(target)}$$
$$Y_{(target)} = k_{0(target)} Y_G$$
$$X_G = \frac{\left[X_{(original)} - X_{0(original)} \right]}{k_{0(original)}}$$
$$Y_G = \frac{Y_{(original)}}{k_{0(original)}}$$

ITRF:

$$\mathbf{r}(t) = \mathbf{r}_0 + \dot{\mathbf{r}}\,(t - t_0)$$

where $\mathbf{r}_0$ and $\dot{\mathbf{r}}$ are the position and velocity respectively at $\mathbf{t}_0$.

Distortion Formulas:

Given: $X = f(\phi, \lambda)$ $\qquad$ $Y = g(\phi, \lambda)$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2};\ m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2 \cos^2 \phi};\ p = \frac{2\left(f_\phi f_\lambda + g_\phi g_\lambda\right)}{R^2 \cos \phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A_p' \ ;$$

$$\sin A_p' = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{\left(f_\lambda g_\phi - f_\phi g_\lambda\right)^2 + \left(f_\phi f_\lambda + g_\phi g_\lambda\right)^2}}$$

Equivalency Condition: $f_\lambda g_\phi - f_\phi g_\lambda = \pm R^2 \cos \phi$

Cauchy-Riemann Equations: $f_\phi = -\frac{1}{\cos \phi} g_\lambda$; $\qquad g_\phi = \frac{1}{\cos \phi} f_\lambda$