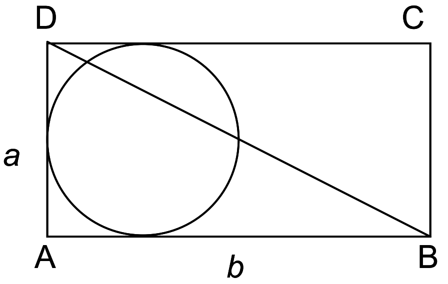
CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C2 – LEAST SQUATRES ESTIMATION & DATA ANALYSIS June 2020

Although calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 9 questions on 3 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	Define or explain briefly the following terms: a) Precision b) Accuracy c) Redundancy of a linear system d) Type II errors in statistical testing e) Internal reliability	10	
2.	Sides *a* and *b* are measured once each as follows: $\boldsymbol{l} = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$ m $\boldsymbol{C}_l = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$ cm^2  a) Estimate the areas of triangle ABD and the circle shown inside the rectangle. b) Estimate the standard deviations of the quantities computed in Part (a). c) Estimate the correlation between the triangle and the circle estimates. d) Discuss the nature of the correlations computed in Part (c).	15	
3.	Consider that the shape of an object is defined by the following equation: $z_i = ax_i^3 + b\sin(y_i)$ where z_i, x_i, y_i are observations with standard deviations $\sigma_{z_i}, \sigma_{x_i}, \sigma_{y_i}$, and *a* and *b* are parameters to be estimated. Assume i = 1, 2, 3. Write the linearized form of this model and derive the required matrices and vectors.	10	
4.	Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station. $\mathrm{C_x} = \begin{bmatrix} 0.000532 & 0.000602 \\ 0.000602 & 0.000838 \end{bmatrix}$ m^2	10	

5.

Prove that $\frac{\sigma}{\sqrt{n}}$ is the standard deviation of the mean value $\bar{x} = \frac{\sum_{i=1}^{n} \ell_i}{n}$, each measurement ℓ_i is made with a standard deviation σ.

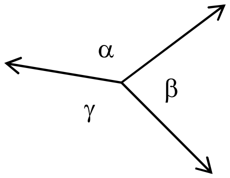
10

6.

Given the angle measurements at a station along with their standard deviations, conduct a conditional least squares adjustment. You are required to compute the following quantities:

a) the estimated residuals
b) the variance-covariance matrix of the estimated residuals
c) the estimated observations
d) the variance-covariance matrix of the estimated observations
e) the estimated variance factor

Angle	**Measurement**	**Standard Deviation**
α	134°38’56”	6.7”
β	83°17’35”	9.9”
γ	142°03’14”	4.3”



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7.

Conduct a parametric least squares adjustment to the same data given in Problem 6. You are required to compute the following quantities:

a) the estimated parameters
b) the variance-covariance matrix of the estimated parameters
c) the estimated difference between α and β
d) the variance of the estimated difference between α and β

10

8.

Given the sample unit variance obtained from the adjustment of a geodetic network $\hat{\sigma}_0^2 = 0.55\,cm^2$ with a degree of freedom $\upsilon = 3$ and the a-priori standard deviation $\sigma_0 = 0.44$ cm, conduct a statistic test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. Provide the major test steps and explain the conclusion.

The critical values that might be required in the testing are provided in the following table:

α	0.001	0.01	0.025	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=3}$	16.26	11.34	9.35	7.82	6.25

10

9.

A baseline of calibrated length (μ) 200.0m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean ($\bar{x}$) and sample standard deviation (s) are calculated from the measurements:

$\bar{x} = 200.5\text{m}$ $\quad\quad$ $s = 0.05\text{m}$

Test at the 95% level of confidence if the measured distance is significantly different from the calibrated distance.

The critical value that might be required in the testing is provided in the following table:

	t_{α}			
Degree of freedom	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

10

Total Marks: 100