

**CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS**

**C-1 MATHEMATICS**

**June 2020**

**Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.**

**Note: This examination consists of 10 questions on 2 pages.**

**Marks**

Q. No

Time: 3 hours

Value   Earned

1.	Find the Taylor polynomial of order 3 generated by $f(x) = \ln x$ at $a = 1$ . Find the Mclaurin series for $f(x) = e^{\frac{x}{2}}$ (a Mclaurin series is a Taylor series at $a = 0$ ).	5 5	
2.	Find the solution $(x, y, z)$ that best fits the five equations $x + z = y$ $3x - y = 2z - 12$ $4x + y + z = -4$ $2z - y = x + 6$ $2y + 2x = 3(z - 3)$ using the least-squares approach.  Find the equation of the parabola $y = ax^2 + bx + c$ that passes through the following three points: $(-2, 40)$ , $(1, 7)$ , $(3, 15)$ .	5 5	
3.	Represent $(\sqrt{3} - i)^{-8}$ as $a + bi$ without the use of a calculator. Show all of your work. Hint: use the polar form $re^{i\theta}$ .  Find the three cube roots of 1 or unity. In other words, find the solution set for $z^3 - 1 = 0$ in the complex numbers.	5 5	
4.	Solve the ordinary differential equation $\frac{d^2y}{dx^2} = y$ for $y$ as a function of $x$ . (Note that this is equivalent to $y'' = y$ .)  Solve the partial differential equation $\frac{\partial^2 f}{\partial x \partial y} = xy$ for $f$ as a function of $x$ and $y$ .	5 5	
5.	A ship leaves Halifax (position $44.67^\circ N, 63.58^\circ W$ ), starting due east and continuing on the great circle. Find its position after it has sailed 1000 nautical miles.  Find its direction after it has sailed 1000 nautical miles from Halifax as above. (Recall that a nautical mile is an angle of one minute along a great circle.)	5 5	

6.	<p>Given the function <math>f(x, y, z) = \cos(xyz)</math>, what is its total derivative?</p> <p>What is the Laplacian <math>\Delta f(x, y, z)</math> of this function? (An alternative notation for <math>\Delta f(x, y, z)</math> is <math>\nabla^2 f(x, y, z)</math>.)</p>	5 5	
7.	<p>A matrix that equals its conjugate transpose is called a Hermitian matrix. Calculate the determinant of the following <math>3 \times 3</math> Hermitian matrix</p> $A = \begin{bmatrix} 2 & 2+i & 4 \\ 2-i & 3 & i \\ 4 & -i & 1 \end{bmatrix}$ <p>For the preceding matrix <math>A</math>, what is <math>A^2</math>?</p>	5 5	
8.	<p>For a function <math>f(x) = 2^{2x}</math>, what is <math>\frac{df(x)}{dx}</math> or <math>f'(x)</math>?</p> <p>Evaluate the definite integral</p> $\int_1^4 \frac{2+x^2}{\sqrt{x}} dx$	5 5	
9.	<p>An arcade uses three different coloured tokens for their game machines. For \$20 you can purchase any of the following mixtures of tokens: 14 gold, 20 silver, and 24 bronze; OR, 20 gold, 15 silver, and 19 bronze; OR, 30 gold, 5 silver, and 13 bronze. What is the monetary value of the bronze token? Use Cramer's rule and show all of your work as if you did not have a calculator.</p> <p>Find all interior angles for the triangle with points <math>P = (-6, -2, -7)</math>, <math>Q = (-2, 1, 6)</math>, <math>R = (-8, 3, -5)</math>.</p>	5 5	
10.	<p>Consider the <math>2 \times 2</math> matrix</p> $A = \begin{bmatrix} 2 & 1 \\ -6 & 7 \end{bmatrix}$ <p>Find its eigenvalues and for each eigenvalue a corresponding eigenvector.</p>	10	
<b>Total Marks:</b>		100	