CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS — October 2019

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 6 questions on 3 pages. **Marks**

Time: 3 hours

Q. No		Value	Earned
1.	Given the UTM map coordinates of points B and C as follows: **Station** / **Easting (m)** / **Northing (m)** B / 564,702.284 / 5,588,965.983 C / 563,836.008 / 5,580,487.376 The other parameters of the map projection are as follows: • Geodetic coordinates of point B are Latitude = 50° 26′ 56.8740″, Longitude = -122° 05′ 19.1800″; • Mean radius of the earth for the region, R_m = 6,382,129.599 m; • Ellipsoidal parameters, a = 6378137.0000 m; e^2 = 0.006694380025; and e'^2 = 0.006739496780. Answer the following: a) Calculate the geodetic azimuth of line BC (to the tenth of a second).	16	
	b) Calculate the ellipsoidal distance between points B and C, and explain the steps and the quantities needed to transform this distance into a mark to mark distance on the ground surface.	8	
	c) Given a 3TM zone (with false Easting of 304,800 m and the scale factor of central meridian of 0.99990) and an overlapping UTM zone having the same central meridian. If the UTM map coordinates of point Q are X = 470,009.001 m, Y = 5,498,349.835 m, calculate the 3TM map coordinates of point Q.	5	
2.	Answer all of the following. a) Explain how an orbital coordinate system is defined (describing the origin and coordinate axes) and describe three of the important parameters needed to convert coordinates in an orbital system to geocentric coordinate system.	7	
	b) What are the names of the two coordinates used in the horizon coordinate system? Describe completely (including the units) how they are measured and why these coordinates for a star depend on the date, time and location of the observer.	5	
	c) What are the names of the curvilinear coordinates of the right ascension and the ecliptic coordinate systems and how are they measured (including their units and zero points of their coordinates)?	6	
3.	Illustrate the UTM and the Polar Stereographic projections in the Northern hemisphere using only well labelled sketches; giving one sketch for the UTM projection and the other sketch for the Polar Stereographic projection. The sketches must show the projections of the Equator, a sample convergence of meridian, Central Meridian, parallels and meridians with the appropriate spacing between the graticule lines clearly illustrated. (Note: To earn full marks, the number of graticule lines must be sufficient to illustrate the spacing between them.)	18	

4.	Answer all of the following. a) Name the parameters of 14-parameter transformation, and explain the purpose of the transformation. b) Describe the relationship between a reference system and a reference frame. c) Describe how isometric latitude compares with the geodetic latitude from the Equator to the Pole and explain what it is and its important uses in map projection.	4 3 4	
5.	Answer all of the following. a) Explain two fundamental roles of time in positioning applications (providing an appropriate example in each). b) Explain the important differences between UT1 and UTC (stating their full names and what distinguishes them). c) Discuss three classes of time scales that are each related to some natural observable phenomenon (stating the phenomena they are related to and how).	4 4 3	
6.	Answer the following. a) The map projection equations relating map projection coordinates (x, y) with the corresponding geographic coordinates (ϕ , λ) can be given as $x = R\lambda \qquad\qquad y = R\sin\phi$ where R is the mean radius of the spherical earth. Determine (showing your mathematical derivations) if the projection is equivalent using two different approaches (the conclusions of the two approaches must be identical). b) Using only the properties of Tissot indicatrices, clearly explain (with necessary equations) if it is possible to have a map projection that is both conformal and equal-area at the same time.	10 3	
		100	

Some potentially useful formulae are given as follows:

$$T - t = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - x_0$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i.

UTM average line scale factor, $\bar{k}_{UTM} = k_0\left[1 + \frac{x_u^2}{6R_m^2}\left(1 + \frac{x_u^2}{36R_m^2}\right)\right]$;

where $x_i = x_i^{UTM} - x_0$; $\quad x_u^2 = x_1^2 + x_1x_2 + x_2^2$

UTM point scale factor, $k_{UTM} = k_0\left[1 + \frac{\Delta x^2}{2R_m^2}\right]$, where $\Delta x = x^{UTM} - x_0$

$$k_{UTM} = k_0\left[1 + \frac{L^2}{2}\cos^2\phi\right]$$

k_0 is scale factor of Central Meridian and x_0 is the False easting value (or 500 000 m) $L = (\lambda - \lambda_0)$ (in radians) for a given longitude λ; and λ_0 is the longitude of the central meridian.

Grid convergence, $\gamma = L\left(1+\frac{L^2}{3}\left(1+3\eta^2\right)\cos^2\phi\right)\sin\phi$

where $\eta^2 = e'^2\cos^2\phi$; $e'^2 = 0.006739496780$; $L = \left(\lambda - \lambda_0\right)$ (in radians); and λ_0 is the longitude of the central meridian.

Geodetic bearing: $\alpha = t + \gamma + (T - t)$

$$Sf = \frac{R_m}{R_m + H_m}$$

Transformation Formulas:

$$X_{(target)} = k_{0(target)} X_G + X_{0(target)}$$
$$Y_{(target)} = k_{0(target)} Y_G$$
$$X_G = \frac{\left[X_{(original)} - X_{0(original)}\right]}{k_{0(original)}}$$
$$Y_G = \frac{Y_{(original)}}{k_{0(original)}}$$

ITRF:

$$\mathbf{r}(t) = \mathbf{r}_0 + \dot{\mathbf{r}}\,(t - t_0)$$

where $\mathbf{r}_0$ and $\dot{\mathbf{r}}$ are the position and velocity respectively at $\mathbf{t}_0$.

Distortion Formulas:

Given: $X = f\left(\phi, \lambda\right)$ $\qquad$ $Y = g\left(\phi, \lambda\right)$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2};\ m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2\cos^2\phi};\ p = \frac{2\left(f_\phi f_\lambda + g_\phi g_\lambda\right)}{R^2\cos\phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A_p' \ ;$$

$$\sin A_p' = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{\left(f_\lambda g_\phi - f_\phi g_\lambda\right)^2 + \left(f_\phi f_\lambda + g_\phi g_\lambda\right)^2}}$$

Equivalency Condition: $f_\lambda g_\phi - f_\phi g_\lambda = \pm R^2\cos\phi$

Cauchy-Riemann Equations: $f_\phi = -\frac{1}{\cos\phi} g_\lambda$; $\qquad$ $g_\phi = \frac{1}{\cos\phi} f_\lambda$