CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS **October 2018**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 7 questions on 3 pages. **Marks**

Time: 3 hours

Q. No		Value	Earned
1.	Answer all of the following: a) Using a well labelled sketch only, illustrate the Transverse Mercator projection in the Northern hemisphere. The sketch must show the projection of the Equator, Central Meridian, parallels and meridians with the appropriate spacing between lines of the graticule clearly illustrated. b) On a UTM projection, the meridian convergence is 1°03′0.0″ at point A with latitude (ϕ = 53° 42′ 30″ N) and longitude (λ = 121° 40′ 00″W), given the longitude of the central meridian $\lambda_0 = 123^\circ$ W. Determine and illustrate with the aid of a well-labeled sketch of the projected meridians what would be the longitude of a point with the same numeric value for convergence, but opposite algebraic sign. Your sketch must illustrate approximately the directions of True North and Grid North, Central Meridian and locations of the two convergences of meridians. c) What are the ellipsoidal (latitude and longitude) coordinates of the points where the meridian convergence values are minimal and maximal in UTM projections? Calculate the meridian convergence values corresponding to those points. (Assume the longitude of the central meridian $\lambda_0 = 123^\circ$ W.) d) Determine the longitude coordinates (along the Equator) of the points where the scale factor distortion is minimal in UTM projections. What is that scale factor distortion? (Assume the longitude of the central meridian $\lambda_0 = 123^\circ$ W.)	8 5 10 5	
2.	Answer all of the following: a) What is a coordinate system? Under what conditions will a coordinate system become a conventional terrestrial reference system (CTRS)? b) Explain two important reasons why surveyors would like to relate their surveys to a coordinate system (as in the Integrated Survey Areas). c) Describe the relationship between a reference system and a reference frame. d) Describe how isometric latitude compares with the geodetic latitude from the Equator to the Pole and explain its important uses in map projection.	5 2 3 4	
3.	Right ascension and Ecliptic systems are two important celestial reference systems approximating an inertial system. Answer the following with regard to this statement: a) Explain why they are inertial systems and justify why one of them can be considered to be more inertial than the other. b) Explain the needs for an inertial system in Geomatics. c) What are the names of the curvilinear coordinates of the right ascension and the ecliptic coordinate systems and how are they measured (including their units and zero points of their coordinates)?	4 2 6	

4.	Answer all of the following: a) Explain two fundamental roles of time in positioning applications (providing an appropriate example in each). b) Explain one important difference in each of the following time systems: TT, UT1, UTC, and GPST (stating their full names and what distinguishes them). c) Discuss three classes of time scales that are each related to some natural observable phenomenon (stating the phenomena they are related to and how).	4 7 3	
5.	The map projection equations relating map projection coordinates (x, y) with the corresponding geographic coordinates (ϕ, λ) can be given as $x = R\lambda \qquad\qquad y = R\sin\phi$ where R is the mean radius of the spherical earth. Determine (showing your mathematical derivations) if the projection is equivalent using two different approaches (the two approaches must give the same conclusion).	10	
6.	Answer all of the following: a) Explain if Z-axis of the International Terrestrial Reference System (ITRS) will move due to polar motion. b) Assume that the (X, Y, Z) axes of a global coordinate system are parallel to the (X, Y, Z) astronomic coordinate system. Explain if the geodetic and astronomic meridian planes for a given point on the Earth's surface are also parallel. Your explanation must demonstrate that you understand what the two meridian planes are. c) The GPS coordinate values, which are usually presented in the (X, Y, Z) geocentric coordinate system, are inconvenient for use by surveyors. Explain 3 important reasons why the coordinate values are inconvenient for use by surveyors.	2 3 3	
7.	Clearly explain the essential differences between the following items and discuss the use of each item in Geomatics. Your explanation and discussion must clearly demonstrate your understanding of each item (while explaining the differences, do not be tempted to state, for example, that one is … and the other is not). a) CGVD28 and CGVD2013 b) Principal scale and Line scale c) Graticule and Grid	6 4 4	
		100	

Some potentially useful formulae are given as follows:

$$T - t = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - x_0$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i.

UTM average line scale factor, $\bar{k}_{UTM} = k_0\left[1 + \frac{x_u^2}{6R_m^2}\left(1 + \frac{x_u^2}{36R_m^2}\right)\right]$;

where $x_i = x_i^{UTM} - x_0$; $x_u^2 = x_1^2 + x_1x_2 + x_2^2$

UTM point scale factor, $k_{UTM} = k_0\left[1 + \frac{\Delta x^2}{2R_m^2}\right]$, where $\Delta x = x^{UTM} - x_0$

$$k_{UTM} = k_0\left[1+\frac{L^2}{2}\cos^2\phi\right]$$

k_0 is scale factor of Central Meridian and x_0 is the False easting value (or 500,000 m)
$L=(\lambda-\lambda_0)$ (in radians) for a given longitude λ; and λ_0 is the longitude of the central meridian.

Grid convergence, $\gamma = L\left(1+\frac{L^2}{3}\left(1+3\eta^2\right)\cos^2\phi\right)\sin\phi$

where $\eta^2 = e'^2\cos^2\phi$; $e'^2 = 0.006739496780$; $L=(\lambda-\lambda_0)$ (in radians); and λ_0 is the longitude of the central meridian.

Geodetic bearing: $\alpha = t + \gamma + (T-t)$

$$Sf = \frac{R_m}{R_m + H_m}$$

Transformation Formulas:

$$X_{(target)} = k_{0(target)}X_G + X_{0(target)}$$
$$Y_{(target)} = k_{0(target)}Y_G$$
$$X_G = \frac{\left[X_{(original)} - X_{0(original)}\right]}{k_{0(original)}}$$
$$Y_G = \frac{Y_{(original)}}{k_{0(original)}}$$

ITRF:

$$\mathbf{r}(t) = \mathbf{r}_0 + \dot{\mathbf{r}}\,(t - t_0)$$

where $\mathbf{r}_0$ and $\dot{\mathbf{r}}$ are the position and velocity respectively at $\mathbf{t}_0$.

Distortion Formulas:

Given: $X = f(\phi,\lambda)$ $\qquad$ $Y = g(\phi,\lambda)$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2};\ m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2\cos^2\phi};\ p = \frac{2\left(f_\phi f_\lambda + g_\phi g_\lambda\right)}{R^2\cos\phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A_p' \ ;$$

$$\sin A_p' = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{\left(f_\lambda g_\phi - f_\phi g_\lambda\right)^2 + \left(f_\phi f_\lambda + g_\phi g_\lambda\right)^2}}$$

Equivalency Condition: $f_\lambda g_\phi - f_\phi g_\lambda = \pm R^2\cos\phi$

Cauchy-Riemann Equations: $f_\phi = -\frac{1}{\cos\phi}g_\lambda$; $\qquad$ $g_\phi = \frac{1}{\cos\phi}f_\lambda$