CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-4 COORDINATE SYSTEMS & MAP PROJECTIONS **March 2019**

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted by the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

Note: This examination consists of 7 questions on 3 pages. **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	Explain the following with regard to map projections. a) Three main uses of first fundamental form and one main use of second fundamental form (clearly demonstrating also that you understand what the first and second fundamental forms are). b) One important use of a reference ellipsoid. c) Developable shapes (including examples) and the significance of a developable shape being secant to the model of the earth during map projection process. d) The main differences between a map and a map projection (the differences must be very clear in order to earn full marks).	5 1 3 2	
2.	Answer all of the following. a) Illustrate the UTM and the Polar Stereographic projections in the Northern hemisphere using only well labelled sketches; give one sketch for the UTM projection and the other sketch for the Polar Stereographic projection. The sketches must show the projections of the Equator, Central Meridian, parallels and meridians with the appropriate spacing between the graticule lines clearly illustrated. (Note: To earn full marks, the number of graticule lines must be sufficient to illustrate the spacing between them.) b) Consider two line segments joining three points A, B, C on an ellipsoidal model of the earth with the given plane bearing BA as $t = 335° \, 54' \, 32''$ and the measured angle (on the ellipsoid) at point B as 210° 37′ 29″. Assume the UTM coordinates of points A and B are known and those of point C are unknown, to be determined. Explain, in point and logical forms (without calculation), how you will determine the plane bearing of line BC (assuming distances BA and BC are very long). c) Given a 3TM zone (with false Easting of 304,800 m and the scale factor of central meridian of 0.99990) and an overlapping UTM zone having the same central meridian. If the UTM map coordinates of point Q are X = 470,009.001 m, Y = 5,498,349.835 m, calculate the 3TM map coordinates of point Q. d) Determine the longitude coordinates (along the Equator) of the points where the scale factor distortion is minimal in UTM projections. What is that scale factor distortion? (Assume the longitude of the central meridian $\lambda_0 = 123^\circ$ W.)	15 6 5 5	
3.	Answer all of the following. a) Explain two important reasons why surveyors would like to relate their surveys to a coordinate system (as in the Integrated Survey Areas). b) Describe the relationship between a reference system and a reference frame. c) Describe how isometric latitude compares with the geodetic latitude from the Equator to the Pole and explain what it is and its important uses in map projection.	2 3 4	

4.	Answer all of the following. a) Explain two fundamental roles of time in positioning applications (providing an appropriate example in each). b) Explain one important difference in each of the following time systems: TT, UT1, UTC, and GPST (stating their full names and what distinguish them). c) Discuss three classes of time scales that are each related to some natural observable phenomenon (stating the phenomena they are related to and how).	4 7 3	
5.	Answer the following. a) The map projection equations relating map projection coordinates (x, y) with the corresponding geographic coordinates (ϕ , λ) can be given as $x = R\lambda$ $\qquad$ $y = R\sin\phi$ where R is the mean radius of the spherical earth. Determine (showing your mathematical derivations) if the projection is equivalent using two different approaches (the conclusions of the two approaches must be identical). b) Using only the properties of Tissot indicatrices, clearly explain (with necessary equations) if it is possible to have a map projection that is both conformal and equal-area at the same time.	10 3	
6.	Answer all of the following. a) Explain if Z-axis of the International Terrestrial Reference System (ITRS) will move due to polar motion. b) Assume that the (X, Y, Z) axes of a global coordinate system are parallel to the (X, Y, Z) astronomic coordinate system. Explain if the geodetic and astronomic meridian planes for a given point on the Earth's surface are also parallel. Your explanation must demonstrate that you understand what the two meridian planes are. c) What are the names of the two coordinates used in the horizon coordinate system? Describe completely (including the units) how they are measured and why these coordinates for a star depend on the date, time and location of the observer. d) Clearly explain the essential differences between CGVD28 and CGVD2013 and discuss their uses in geomatics (demonstrating in your explanation and discussion your understanding of the meaning of each term).	2 3 5 6	
7.	What is the purpose of 14-parameter transformation? Name the parameters of the 14-parameter transformation. What are the two underlying assumptions made in the transformation function?	6	
		100	

Some potentially useful formulae are given as follows:

$$T - t = \frac{(y_2 - y_1)(x_2 + 2x_1)}{6R_m^2}$$

where $y_i = y_i^{UTM}$; $x_i = x_i^{UTM} - x_0$; R_m is the Gaussian mean radius of the earth; and x_i^{UTM} and y_i^{UTM} are the UTM Easting and Northing coordinates respectively, for point i.

UTM average line scale factor, $\bar{k}_{UTM} = k_0\left[1 + \frac{x_u^2}{6R_m^2}\left(1 + \frac{x_u^2}{36R_m^2}\right)\right]$;

where $x_i = x_i^{UTM} - x_0$; $x_u^2 = x_1^2 + x_1x_2 + x_2^2$

UTM point scale factor, $k_{UTM} = k_0\left[1+\frac{\Delta x^2}{2R_m^2}\right]$, where $\Delta x = x^{UTM} - x_0$

$$k_{UTM} = k_0\left[1+\frac{L^2}{2}\cos^2\phi\right]$$

k_0 is scale factor of Central Meridian and x_0 is the False easting value (or 500 000 m)
$L = (\lambda - \lambda_0)$ (in radians) for a given longitude λ; and λ_0 is the longitude of the central meridian.

Grid convergence, $\gamma = L\left(1+\frac{L^2}{3}\left(1+3\eta^2\right)\cos^2\phi\right)\sin\phi$

where $\eta^2 = e'^2\cos^2\phi$; $e'^2 = 0.006739496780$; $L = (\lambda - \lambda_0)$ (in radians); and λ_0 is the longitude of the central meridian.

Geodetic bearing: $\alpha = t + \gamma + (T - t)$

$$Sf = \frac{R_m}{R_m + H_m}$$

Transformation Formulas:

$$X_{(target)} = k_{0(target)} X_G + X_{0(target)}$$
$$Y_{(target)} = k_{0(target)} Y_G$$
$$X_G = \frac{\left[X_{(original)} - X_{0(original)}\right]}{k_{0(original)}}$$
$$Y_G = \frac{Y_{(original)}}{k_{0(original)}}$$

ITRF:

$$\mathbf{r}(t) = \mathbf{r}_0 + \dot{\mathbf{r}}\,(t - t_0)$$

where $\mathbf{r}_0$ and $\dot{\mathbf{r}}$ are the position and velocity respectively at $\mathbf{t}_0$.

Distortion Formulas:

Given: $X = f(\phi,\lambda)$ $\qquad Y = g(\phi,\lambda)$

$$m_1^2 = \frac{f_\phi^2 + g_\phi^2}{R^2};\ m_2^2 = \frac{f_\lambda^2 + g_\lambda^2}{R^2\cos^2\phi};\ p = \frac{2\left(f_\phi f_\lambda + g_\phi g_\lambda\right)}{R^2\cos\phi}$$

$$\frac{d\Sigma'}{d\Sigma} = m_1 \times m_2 \sin A_p' \;;$$

$$\sin A_p' = \frac{f_\lambda g_\phi - f_\phi g_\lambda}{\sqrt{\left(f_\lambda g_\phi - f_\phi g_\lambda\right)^2 + \left(f_\phi f_\lambda + g_\phi g_\lambda\right)^2}}$$

Equivalency Condition: $f_\lambda g_\phi - f_\phi g_\lambda = \pm R^2\cos\phi$

Cauchy-Riemann Equations: $f_\phi = -\frac{1}{\cos\phi} g_\lambda$; $\qquad g_\phi = \frac{1}{\cos\phi} f_\lambda$