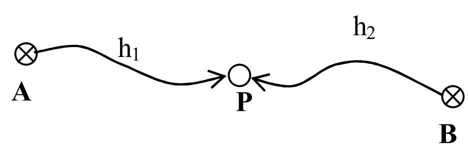
CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-2 LEAST SQUARES ESTIMATION & DATA ANALYSIS **March 2019**

Note: This examination consists of 9 questions on 3 pages. **Marks**

Q. No Time: 3 hours Value Earned

<table>
<tr><th>Q. No</th><th></th><th>Value</th><th>Earned</th></tr>
<tr><td>1.</td><td>Explain briefly the following (each 2.5 marks):
a) Standard deviation
b) Circular error probability
c) Root mean square error
d) Correlation coefficient
e) Type II error in statistical testing
f) Redundancy of a linear system</td><td>15</td><td></td></tr>
<tr><td>2.</td><td>Given the cofactor matrix Q of the horizontal coordinate (x, y) of a survey station and the unit variance $\hat{\sigma}_0^2 = 2\,\text{cm}^2$, calculate the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station.
$$Q = \begin{bmatrix} 5.32 & 6.02 \\ 6.02 & 8.38 \end{bmatrix}$$</td><td>10</td><td></td></tr>
<tr><td>3.</td><td>Given a leveling network below where A and B are known points, h_1 and h_2 are two height difference measurements with standard deviation of σ_1 and σ_2, respectively and $\sigma_1 = 2\ \sigma_2$. Determine the value of σ_1 and σ_2 so that the standard deviation of the height solution at point P using least squares adjustment is equal to 2mm.

</td><td>10</td><td></td></tr>
<tr><td>4.</td><td>Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$:
$$C_\ell = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$
and two functions of $\ell : x = \ell_1 + \ell_2$ and $y = 3\ell_1$, determine $C_{xy}, C_{x\ell}, C_{y\ell}$</td><td>10</td><td></td></tr>
</table>

5. Given the following mathematical model

$$f(\ell, x) = 0 \qquad C_\ell \quad C_x$$

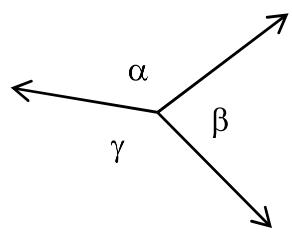
where f is the vector of mathematical models, x is the vector of unknown parameters and C_x is its variance matrix, ℓ is the vector of observations and C_ℓ is its variance matrix.

a) Linearize the mathematical model — 5
b) Formulate the variation function — 5

6. Given the angle measurements along with their standard deviations, conduct a conditional least squares adjustment. You are required to compute the following quantities: — 15

a) the estimated residuals
b) the variance-covariance matrix of the estimated residuals
c) the estimated observations
d) the variance-covariance matrix of the estimated observations
e) the estimated variance factor

Angle	**Measurement**	**Standard Deviation**
α	134°38’56”	6.7”
β	83°17’35”	9.9”
γ	142°03’14”	4.3”



7. Conduct a parametric least squares adjustment to the same data given in Problem 6. You are required to compute the following quantities: — 10

a) the estimated parameters
b) the variance-covariance matrix of the estimated parameters
c) the estimated difference between α and β
d) the variance of the estimated difference between α and β

8. Given the sample unit variance obtained from the adjustment of a geodetic network $\hat{\sigma}_0^2 = 0.55\,\text{cm}^2$ with a degree of freedom $\upsilon = 3$ and the a-priori standard deviation $\sigma_0 = 0.44\,\text{cm}$, conduct a statistical test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$.

The critical values that might be required in the testing are provided in the following table:

α	0.001	0.01	0.025	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=3}$	16.26	11.34	9.35	7.82	6.25

where $\chi^2_{\alpha,\ \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=3}}^{\infty} \chi^2(x)dx$.

Marks: 10

9. The following residual vector $\hat{r}$ and estimated cofactor matrix $Q_{\hat{r}}$ were computed from a least squares adjustment using independent observations with a standard deviation of $\sigma_0 = 1.5$ mm:

$$\hat{r} = \begin{bmatrix} 4 & 2 & -3 & 10 \end{bmatrix} \qquad \text{(mm)}$$

$$Q_{\hat{r}} = \begin{bmatrix} 15 & 1 & 3 & -2 \\ 1 & 7 & -1 & 3 \\ 3 & -1 & 4 & -1 \\ -2 & 3 & -1 & 2 \end{bmatrix} \qquad (\text{mm}^2)$$

Given that a global test has been rejected with a significance level of α = 0.04, conduct further tests to identify which observation(s) may contain an outlier. The critical values that might be required in the testing are provided in the following table:

α	0.001	0.002	0.003	0.004	0.005	0.01	0.05
K_α	3.09	2.88	2.75	2.65	2.58	2.33	1.64

where K_α is determined by the equation $\alpha = \int_{K_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$.

Marks: 10

Total Marks: 100