**Canadian Board of Examiners for Professional Surveyors**
**Core Syllabus Item**
**C 4: COORDINATE SYSTEMS AND MAP PROJECTIONS**

**Study Guide:**

A collection of formulae are provided with the examination questions when necessary.

1. With reference to:

   Hradilek, L. and A.C. Hamilton [1973]. Systematic Analysis of Distortions in Map Projections. Geodesy and Geomatics Engineering Department Lecture Notes No. 34, University of New Brunswick, Fredericton, Canada. [*Available via <http://www2.unb.ca/gge/HomePage.php> under "Publications": "Lecture Notes" or via* <http://www2.unb.ca/gge/Pubs/LN34.pdf>].

   Q1.1. What are the main problems associated with map projections? (see Introduction section)

   Q1.2. Describe the shapes and sizes of Tissot indicatrices with regard to conformality, equivalency (equal-area) and equidistancy. (see Chap. II).

   Q1.3. The functional relationships between the mapping coordinates (X, Y) and the geodetic coordinates (Latitude = $\phi$, Longitude = $\lambda$) for a simple cylindrical projection are

   $X = R\phi \qquad\qquad Y = R\lambda$

   See Chapter III, p.16):

   a. What are the directions of the extreme distortions?

   b. Is the projection equal-area?

   c. What is the area distortion factor?

   d. Is the projection conformal?

   e. What is the maximum distortion angle?

2. With reference to:

   Krakiwsky, E.J. [1973]. Conformal Map Projections in Geodesy. Department of Geodesy and Geomatics Engineering Lecture Notes No. 37, University of New Brunswick, Fredericton, Canada. [*Available via <http://www2.unb.ca/gge/HomePage.php> under "Publications": "Lecture Notes" or via* <http://www2.unb.ca/gge/Pubs/LN37.pdf>].

   Q2.1. Q1: What is an isometric plane? Explain the geometric interpretation of the projected parallels and meridians on this plane. (see sec. 4.1)

   Q2.2. Q2: Explain the variations in the arc-to-chord correction with respect to Transverse Mercator projections. (see sec. 4.5, 6)

   Q2.3. Q3: Explain the variations of convergence angles with respect to Transverse Mercator projections. Are the convergence angles affected by the length or the direction of the lines connecting any two stations? Explain. (see sec.6.2)

Q2.4. Using a well labelled sketch only, illustrate the appearance of Transverse Mercator projection in the Northern hemisphere. The sketch must show the projected **Equator**, **Central Meridian**, **parallels** and **meridians** with the appropriate relationship between the lines of the graticule clearly represented. (see sec. 6.4)

Q2.5. What is footpoint latitude? (see sec. 6.5)

Q2.6. Name the different types of map projections used by the Canadian government. Describe the characteristics of each projection with regard to the exactness of distance, area and angle measurements. (see sec. 6.8, 6.9, 8.2)

Q2.7. The scale factor ($k$) at any given point ($\phi$, $\lambda$) on a UTM projection can be determined using the following formula:

$$k = k_0 \left[ 1 + \ell^2 \frac{\cos^2 \phi}{2} \right]$$

where $\ell = \lambda_{CM} - \lambda$, expressed in radians; $\lambda_{CM}$ is the longitude of the central meridian; $k_0$ is the scale factor at the central meridian; and $\phi$ and $\lambda$ are the latitude and longitude values of the given point. At what distance (to one decimal degree) away from the central meridian, say along the equator, is the UTM scale distortion equal to zero?

Answer:

1.6°; see sec. 6.8

Q2.8. Using a well labelled sketch only, illustrate the Mercator projection in the Northern hemisphere. The sketch must show the projected **Equator**, **Central Meridian**, **parallels** and **meridians** with the appropriate relationship between the lines of the graticule clearly represented. (see sec. 5)

Q2.9. Explain the appearance of a Mercator projection of an area with large extent in the north-south direction, with regard to scale distortion, meridian convergence and shape. (see sec. 5)

Q2.10. Using a well labelled sketch only, illustrate the appearance of the Lambert Conformal Conic projection in the Northern hemisphere. The sketch must show the projected **Equator**, **Central Meridian**, **parallels** and **meridians** with the appropriate relationship between the lines of the graticule clearly represented. (see sec. 7)

Q2.11. Explain the variations of convergence angles with respect to Lambert Conic projections. Are the convergence angles affected by the length or the direction of the lines connecting any two stations? Explain. (see sec. 7)

Q2.12. Why do the surveyors compute geodetic positions on a conformal projection plane instead of, for example, on an equal-area or an equidistant map projection plane? (see Chap. 10)

Q2.13. Give one advantage and one disadvantage of geodetic coordinates over map projection coordinates. (see Chap. 10)

Q2.14. You are given the geodetic distance of 1987.62 m and the geodetic azimuth, from a monument with UTM coordinates, to a point B. The geodetic latitude and longitude of the observation point are known. Answer the following questions as completely as possible; listing all other information you may need in each case.

a. Describe how you would manually reduce the geodetic azimuth to a grid azimuth. (see sec. 6.2, 11, 14.1)

b. Describe how you would manually reduce the geodetic distance to a grid distance. (see sec. 11.3, 14.1.3)

3. With reference to:

Krakiwsky, E.J. and D.E. Wells [1971]. Coordinate Systems in Geodesy. Department of Geodesy and Geomatics Engineering Lecture Notes No. 16, University of New Brunswick, Fredericton. [*Available via <http://www2.unb.ca/gge/HomePage.php> under "Publications": "Lecture Notes" or via <http://www2.unb.ca/gge/Pubs/LN16.pdf>*].

Q3.1. A time system is defined by specifying an interval and an epoch. Explain the interval and epoch of the two systems of time such as universal time and sidereal time. (see sec. 1.2)

Q3.2. What is the difference between the Geocentric Geodetic System and the Conventional Terrestrial System? Why is Geocentric Geodetic System commonly used? (see sec. 2.1 & 2.3)

Q3.3. Name two kinds of terrestrial topocentric coordinate system. How are the z-axes of the two systems defined? How are the two z-axes related, explaining why they are not aligned in the same direction in space? (see sec. 2.4)

Q3.4. What basic assumptions are made about the Earth and the stars with regard to the definition of the celestial sphere? (see sec. 3)

Q3.5. Explain the major use of each of the following coordinate systems: Ecliptic System, Right Ascension System, Hour Angle System, Horizon System. (see sec. 3)

Q3.6. What are the differences between the Horizon System and the Local Astronomic System? (see sec. 2.4.1 & 3.4)

Q3.7. How is the Orbital Coordinate System specified? (see sec. 4.2)

Q3.8. Explain how the Orbital System can be linked to the Conventional Terrestrial (Average Terrestrial) System. (see sec. 4.3)

4. With reference to:

*Natural Resources Canada [2014]. "Geodetic Reference Systems". Available via http://www.nrcan.gc.ca/earth-sciences/geomatics/geodetic-reference-systems/10781*.

Q4.1. Explain what constitutes the realization of the Canadian Spatial Reference System (CSRS).

Q4.2. Clearly explain one important difference between CGVD28 and CGVD2013 (do not be tempted to state, for example, that one is … and the other is not).

Q4.3. Explain the essential differences between NAD83 original and NAD83(CSRS) datums.

Q4.4. When the heights of benchmarks from a differential levelling (based on CGVD28) are compared with the orthometric heights of the same benchmarks based on GPS / geoid model (CGG2000) approach, they may differ. Explain what could contribute to the difference.

Q4.5. Answer the following as completely as possible:

a. What is a vertical datum?

b. What is the difference between the published CGVD28 orthometric height and ellipsoidal height derived from GPS?

c. In practice, what would you do to transform from one to another?

5. With reference to:

Snyder, J.P. [1987]. *Map Projections – A Working Manual.* U.S.A. Geological Survey Professional Paper 1395. United States Government Printing Office, Washington. Chapters 1-8, 14, 15, 21 and Appendix A. [available via < http://pubs.er.usgs.gov/djvu/PP/PP_1395.pdf>].

Q5.1. What is a map projection? Why do different map projections exist? (see chap. 1)

Q5.2. What are developable shapes? Name them, explaining why the shapes are considered developable? (see chap. 1)

Q5.3. Explain what you understand by the term conformality in map projections. (see chap. 1)

Q5.4. What are your considerations in choosing the best map projection for a region? (see chap. 1)

Q5.5. Given a map projection software application, explain how you would assess the accuracy of the output. (chaps. 1 & 4)

Q5.6. Explain Tissot Indicatrix with respect to its clear meaning and main application or use. (see chap. 4).

Q5.7. Classify map projections based on the following: aspects; projection surfaces (or developable shapes), distortion properties. (see chap. 6)

Q5.8. Give two important uses and one important limitation of Mercator projection. (see chap. 7)

Q5.9. See Numerical Examples on p.269-271 & p.296-298.

6. With reference to:

Thomson, D. B., E. J. Krakiwsky, R. R. Steeves (1977). *A manual for Geodetic Coordinate Transformations in the Maritime Provinces*, Geodesy and Geomatics Engineering Department Technical Report No. 48, University of New Brunswick, Fredericton, Canada. [*Available via < http://www2.unb.ca/gge/HomePage.php > under "Publications": "Technical Reports" or via or via <http://www2.unb.ca/gge/Pubs/TR48.pdf>*].

Q6.1. Explain the parameters that must be determined in defining a geodetic datum by a classical method. (see Chap. 3)

Q6.2. Name 8 of the main parameters required in any geodetic map projection. (see sec. 6.1, 7.1 or 8.1)

Q6.3. Considering the shape and size of Nova Scotia, what map projection would you recommend to the Province in order to achieve a minimum distortion on a map with a scale of 1:10,000? Explain your answer with regard to the expected distortion properties including suggested scale factor range, developable surface, aspect, suitable earth model (s), and type of contact (secant or tangent). (see sec. 8)

*<u>Notes:</u> MTM is an acronym for Modified Transverse Mercator, which is usually used in relation to a projection that is similar to UTM but having a zone width of 3˚. Some groups prefer to call it 3˚ Transverse Mercator (3˚TM, 3TM, or 3˚MTM). The Province of Nova Scotia uses the term 3˚MTM while the Province of Alberta seems to prefer 3TM. Both cases, however, refer to the same thing. The 3TM (or MTM) and UTM projections are mainly different in the choice of scale factor at the central meridian and the location of the central meridian.*

Q6.4. See Numerical Examples 5.3, 6.3, 7.3 & 8.3.

7. With reference to:

Thomson, D. B., M. P. Mephan, and R. R. Steeves (1998). <u>*The Stereographic Double Projection,*</u> Geodesy and Geomatics Engineering Department Technical Report No. 46, University of New Brunswick, Fredericton, Canada. Chapters 1 - 4. [*Available via < http://www2.unb.ca/gge/HomePage.php > under "Publications": "Technical Reports" or via <[http://www2.unb.ca/gge/Pubs/TR46.pdf](http://www2.unb.ca/gge/Pubs/TR46.pdf)>*].

Q7.1. Considering the shape and size of New Brunswick, what map projection would you recommend to the Province in order to achieve a minimum distortion on a map with a scale of 1:10,000? Explain your answer with regard to the expected distortion properties including suggested scale factor range, developable surface, aspect, suitable earth model (s), and type of contact (secant or tangent).

Q7.2. Considering the shape and size of Prince Edward Island, what map projection would you recommend to the Province in order to achieve a minimum distortion on a map with a scale of 1:10,000? Explain your answer with regard to the expected distortion properties including suggested scale factor range, developable surface, aspect, suitable earth model (s), and type of contact (secant or tangent).

8. With reference to:

Torge, W. and J. Müller [2012]. <u>*Geodesy*</u>. 4th edition, Walter de Gruyter GmbH & Co. KG, Berlin/Boston, ISBN 978-3-11-020718-7, e-ISBN 978-3-11-025000-8. [Reference systems and Reference Frames (Chapter 2); Section 3.2 (relating to Natural coordinates) ; Section 3.4 (The geoid); Chapter 4 (relating to Coordinate systems and Geodetic reference systems); Section 5.2 (relating to orbital and equatorial systems); Section 5.3 (relating to Geodetic astronomy); Sections 6.2 -6.3 (relating to 3D coordinate systems, Geodetic datum and the relevance of ellipsoid); and Sections 7.1-7.3 (relating to NAD27, NAD83, NGVD29, 3D global geodetic reference).

Torge, W. [2001]. <u>*Geodesy*</u>. 3rd edition, Walter de Gruyter, N.Y. , ISBN 3-110-17072-8. [Reference systems (18-44); Natural coordinates (64-66); The Geodetic Earth Model (91-102)].

Q8.1. Explain the fundamental role that time plays in Geomatics. (see sec. 2.2)

Q8.2. Explain what you understand by inertial time, solar time, and sidereal time. How are they related and what are their important uses? (see sec. 2.2)

Q8.3. What is an inertial system? Explain the need for it and describe an approximation to an inertial system. (see sec. 2.3 & 2.4].

Q8.4. Explain the differences between the following: spatial geodetic coordinate system and the Earth-fixed geocentric system; Local geodetic system and local astronomic system? (see sec. 2.3.3, 2.5 & 4.1.3)

Q8.5. Describe the Terrestrial Reference System (TRS). How is it defined (giving the orientations of X, Y, Z axes in space) and realized (giving an example)? Why is a reference time epoch important in this system? (see sec. 2.3.3 & sec. 2.4.2)

Q8.6. Explain the relationship between a reference frame and a reference system, giving one specific example of each. (see sec. 2.4)

Q8.7. Give the characteristics of a Conventional Terrestrial Reference System. (see sec. 2.3.3)

Q8.8. Describe the International Celestial Reference System (ICRS). How is it realized? (see sec. 2.3.1 & 2.4)

Q8.9. Describe how a system of natural coordinates can be defined, and explain how the natural coordinates can be determined by measurements. (see text, sec. 3.2 & 3.2.3)

Q8.10. Describe the Local Astronomic systems and why they are important in Geomatics. (see text, sec. 2.3, 2.5, 3.2 & 5.3)

Q8.11. What are the differences between the local astronomic and global geocentric systems? Give the mathematical relationship between the two systems. (see sec. 2.3, 2.5, 3.2 & 5.3)

Q8.12. What is a geodetic earth model and why is it important in Geomatics? (see sec. 4.0)

Q8.13. What is a geodetic datum? Explain what constitutes the realization of a geodetic datum. (see sec. 6.2.2)

9. Additional Study Questions

Q9.1. On a UTM projection, the meridian convergence for point B with latitude 53° 42′ 28″ N and longitude -112° 18′ 29″ (or W) is calculated as -1° 03′ 15.73″. Would this convergence change in a 3TM zone with the same central meridian as that used for the UTM? If so, why? What would be the longitude of a point with the same numeric value for the convergence, but opposite algebraic sign? In what UTM zone is the point located? (Central meridian is -111° (or W))

Answer:

No, $\lambda$ = -109°41′ 31″, UTM Zone 12

Q9.2. If a 3TM zone (having no false eastings and no false northings) and a UTM zone have the same central meridian, what is the UTM easting of a point with a 3TM easting of 113,660.42 m (assuming the 3TM scale factor of the central meridian is 0.99990)?

Answer:

613,626.32 m; refer also to sec. 6.8-6.9 in Krakiwsky[1973].

Q9.3. Using well labelled sketches only, illustrate the Mercator and the Transverse Mercator projections in the Northern hemisphere; give one sketch for the Mercator projection and the other sketch for the Transverse Mercator projection. The sketches must show the projections of the **Equator**, **Central Meridian**, **parallels** and

**meridians** with the appropriate relationship between the lines of the graticule clearly illustrated.

Answer:

See sec. 5 & 6 in Krakiwsky[1973]; and sec. 7 & 8 in Snyder [1987].

Q9.4. Considering the shape and size of New Brunswick, what map projection would you recommend to the Province in order to achieve a minimum distortion on a map with a scale of 1:10,000? Explain your answer with regard to the expected distortion properties including suggested scale factor range, developable surface, aspect, suitable earth model (s), and type of contact (secant or tangent).

Answer:

See Thomson, et al. [1998] and Thomson, et al.[1977]

Q9.5. The Canadian base maps are produced on a series of projections for each individual province. This is done in order to keep the scale factor distortion in each province to a very low level. A company has just acquired map data and wishes to use only one projection, in order to avoid having discontinuities. Area is not important, as this will be an attribute on the database, but the shapes of features should not have any great distortion. Design a suitable projection, given the range of latitude as 41°N to 84°N and the range of longitude as 52°W to 141°W.

Answer:

See sec. 7 in Krakiwsky[1973]; and sec. 15 in Snyder [1987]..

Q9.6. On a UTM projection, calculate the meridian convergence (to the nearest arc minute) for point A with latitude ($\phi$ = 53° 42′ 28″ N) and longitude ($\lambda$ = 112° 18′ 29″W), given the longitude of the central meridian, $\lambda_0 = 111^\circ$ W and the following equation:

$$\gamma = \ell\left(1 + \frac{\ell^2}{3}\left(1 + 3\eta^2\right)\cos^2\phi\right)\sin\phi$$

where $\gamma$ is the meridian convergence, $\eta^2 = e'^2\cos^2\phi$ , $e'^2 = 0.006739496780$ and $\ell = \lambda - \lambda_0$ .

Answer:

-1°03′16″

Q9.7. The scale factor ($k$) at any point ($x$, $y$) on a UTM projection can be determined using the following formula:

$$k = k_0\left[1 + \frac{(x - x_0)^2}{2R^2}\right]$$

where $k_0$ and $x_0$ are the scale factor and the false Easting coordinate at the central meridian, respectively, and R is the mean radius of the earth. In a large-scale cadastral mapping of a region (with 360 km East-West extent), a scaling accuracy ratio of 1/10,000 is required and a modified Transverse Mercator projection (similar to UTM) is to be used. The radius of the earth in the region can be taken as 6,371 km.

a. Determine the number of zones (showing the computational steps followed) and the scale factor (to 6 decimal places) to be used at the central meridian so that the scaling accuracy ratio remains within 1/10,000.

b. What is the distance between the two secant lines in a zone while still maintaining the scaling accuracy of 1/10,000 in the region?

c. If a single zone is used for the whole mapping region, what would the worst scaling accuracy ratio for the zone be, assuming the scale factor determined in (a) is adopted for the central meridian?

Answer:

a. Two zones of about 250 km and 110 km

b. 180 km

c. $\frac{1}{3,000}$