

**Canadian Board of Examiners for Professional Surveyors**  
**Core Syllabus Item**  
**C 3: ADVANCED SURVEYING**  
**Study Guide**

Critical values of the  $\chi^2$  distribution and a collection of formulae are provided with the examination questions.

**Study Questions 1 to 6**

**Refer to Ogundare [2016]** for similar questions:

**SQ1:** The effects of lateral refraction can be quantified, more to recognize when conditions should be avoided rather than to apply as a correction. Along the south face of a block of buildings, temperature readings were taken at the wall surface and at 1 m away. The average values were 35 C and 30 C, respectively. A traverse around the block had to be run with the lines offset by 0.5 m from the building faces. The block is 300 m square. By how much is the effect of refraction along this one side likely to contaminate the misclosure of the block, assuming standard pressure?

ans: 4' 20" = 260" (130" smaller at each of the SE and SW corners)

**SQ2:** EODMI are designed to be used at an average temperature, pressure, and relative humidity [e.g., 12 C, 1013.25 mb, 60%].

a: If the accuracy is to be maintained to within 2 ppm, what would be the limits on the variation of i: temperature or ii: pressure from the design values?

ans: i:  $10.0\text{ C} \leq t \leq 14.0\text{ C}$ , or ii:  $1007.3 \leq p \leq 1019.2\text{ mb}$

b: If the accuracy is to be maintained to within 2 ppm, what would be the limits on the variation of temperature and pressure, simultaneously, from the design values?

ans:  $10.6\text{ C} \leq t \leq 13.4\text{ C}$  and  $1009.0 \leq p \leq 1017.5\text{ mb}$

**SQ3:** Geodetic measurements are usually made to determine "absolute" position. They can be repeated at a later time to determine change in position over that period.

Geotechnical instrumentation provides relative changes. Explain how geodetic and geotechnical observations can complement each other in the monitoring of a sensitive structure: a: for horizontal or planimetric deformation; b: for vertical deformation; and c: for three dimensional deformation.

**SQ4:** The azimuth of a reference line can be determined using a gyrotheodolite or a gyro attachment. Commonly underground activity, such as in mining, is described in a local

coordinate system. Explain what corrections need to be applied to a gyro azimuth, and under what circumstances, in order to for it to be used in the coordinate system of the activity.

**SQ5:** Most typical land surveying activity employs plane surveying. Explain the circumstances under which it is necessary to be concerned about the geodetic aspects of surveying.

**SQ6:** Suggest, with examples, the achievable limits on the precision and on the accuracy of geodetic surveys for deformation monitoring.

### Study Question 7

The maximum allowable angular misclosure in a traverse of  $n_\beta$  angles is stated as  $M_\beta$  at 99%.

- a) Determine the standard deviation,  $\sigma_\beta$ , of each of the  $n_\beta$  angles, considering that each would contribute equally to the actual misclosure  $m_\beta$ .

$$\text{Ans : } \sigma_\beta \leq \frac{M_\beta}{2.57\sqrt{n_\beta}} \text{ by formulating the relationship between traverse angles and}$$

misclosure and propagating the error.

- b) If the average from  $n_s$  sets of an angle has a standard deviation of  $\pm \sigma_\beta$ , determine the allowable discrepancy,  $\delta_s$ , between individual sets that would be used as a quality check at the time of observation.

$$\text{Ans: } \delta_s \leq \sigma_\beta \sqrt{2n_s} \text{ by propagating through the average back into the individual angle and then to the discrepancy.}$$

[refer to Ogundare, 2016]

### Study Question 8

Monitoring a structure over an extended period of time involves the repetition of observations that may be used to determine the geometric state at a particular epoch  $t_i$  ( $\mathbf{x}_{ti}$ ), and then again at  $t_j$  ( $\mathbf{x}_{tj}$ ), or to determine how the geometric state has changed over an interval,  $\Delta t_{ij}$  ( $\Delta \mathbf{x}_{ij}$ ). Explain how repeated geodetic observations might be used to describe the “tilt” of a structure. With some explanation, compare the achievable precision of a geodetic observable with a corresponding geotechnical observable [e.g., creating a “long base” tiltmeter between two floor level benchmarks, 20 m apart, by repeated levelling using a Wild N3 (42X, setting accuracy:  $\pm 0.2''$ , micrometer: 0.1 mm), set up in between, and invar double scale staves].

$$\text{ans: } \sigma_{\Delta H} \leq \pm 0.035 \text{ mm}; \sigma_{\delta \Delta H} \leq \pm 0.049 \text{ mm}; \sigma_t \leq \pm 0.50''$$

example done in Chrzanowski [1993]; refer to similar example in Ogundare [2016].

**Reference:** Chrzanowski, A. [1993]. “Modern Surveying Techniques for Mining and Civil Engineering.” Chapter 33 in J.A. Hudson (edit) *Comprehensive Rock Engineering, v.3 Rock Testing and Site Characterization* Pergammon Press, ISBN 0-08-042066-4, p. 773 - 809.

### Study Question 9

During a city survey, it was necessary to run a traverse line at  $\sim 0.4$  m away from the south face of several buildings over a distance of  $\sim 270$  m. The temperature at the building face was about 40 C and, at  $\sim 1$  m away, was 30 C. This line is one of the four traverse lines supposedly closing around the block.

Explain what might affect the angular misclosure and suggest by how much and whether the misclosure would appear to be larger or smaller than it would be without the influence.

What would you do about it?

ans: The line of sight at the SE and SW corners curves toward the building so each angle is smaller by  $0^{\circ}03'49''$  [Chrzanowski, 1993] so the misclosure is smaller by twice that. [Refer also to Ogundare, 2016.]

### Study Question 10

Canadian Special Order Levelling procedures require that “... difference between backsight and foresight distances at each set-up and their total for each section not to exceed 5 m ...” with maximum lengths of sight of 50 m. Normally, invar double scale rods and a level [ $M \geq 40X$ , sensitivity  $\leq 10''/\text{div}$ ] with parallel plate micrometer are used. Explain how well the lengths of sight would have to be determined [i.e.,  $\sigma_s$ ]? How would they be measured? Interpret “not to exceed” as being at 99%.

ans: Each length of sight must be  $\pm 1.37$  m by propagation back from the discrepancy. A table in Anderson and Mikhail [1998] shows careful pacing  $\pm 1/100$ , stadia  $\pm 1/300$ , taping  $\pm 1/1000$ . These can be applied to the length of sight to see whether the method is appropriate, keeping in mind the conditions, esp. the nature of the terrain. [Refer also to Ogundare, 2016]

**Reference:** Anderson, J.M., and E.M. Mikhail [1998]. *Surveying: Theory and Practice* McGraw-Hill Book Company, ISBN 0-07-015914-9 hardcover.

### Study Question 11

For visible and near infra-red radiation and neglecting the effects of water vapour

pressure, the refractivity correction,  $\Delta N_i$ , can be determined by

$$\Delta N_i = N_D - N_i = 281.8 - \left[ \frac{0.29065 p}{1 + 0.00366086 t} \right]$$

The meteorological correction is in the sense that  $s = s' + c_{\text{met}}$ , with  $c_{\text{met}} = \Delta N_i s'$ .

- a) Temperature and pressure are to be measured at each end of a 1600 m distance, the refractivity correction at each end will be calculated, and the average value of  $\Delta N_i$  will be used to determine the meteorological correction,  $c_{\text{met}}$ . The instrument being used has a design  $n_D = 1.0002818$  [so that  $N_D = 281.8$ ] and the average temperature and pressure during the measurements are expected to be  $+30^\circ\text{C}$  and 1000 mb. What would be the largest values of  $\sigma_t$  and  $\sigma_p$  that, together with equal contribution to  $\sigma_{\Delta N}$ , would result in a meteorological correction that would contribute uncertainty of no more than 2 ppm to the corrected distance?

ans:  $\pm 2.29^\circ\text{C}$ ,  $\pm 7.56$  mb, propagating through the above equation and the average to  $c_{\text{met}}$ .

- b) If the accuracy [not “precision”] of a distance is to be degraded by no more than 2 ppm as a result of the meteorological correction, what concerns would you have in deciding on equipment and procedures for measuring temperature and pressure?

ans: if the total of systematic influences is to be no more than 2 ppm, explain what testing and calibration would be needed and what corrections would have to be applied. Refer to Section 7.6 [Ogundare, 2016]

### Study Question 12

- a) The transfer of position (x,y “horizontal”) and orientation (azimuth in the x,y plane) from the surface via a single shaft to a tunnel or an adit can be done using either a pair of plumb lines or by optical methods. Briefly explain the advantages and limitations of each method [plumb lines, optical] for i) position transfer and for ii) orientation transfer.
- b) The transfer of elevation from surface benchmarks via a single shaft to underground stations can be done using a steel tape or an EODMI (total station).
- i) Briefly explain the advantages and limitations of each method.
  - ii) With a brief explanation, compare the corrections applied to the tape in elevation transfer to the corrections normally applied when a tape is used to determine horizontal distances.

Refer to Section ch. 12 [Ogundare, 2016]

### Study Question 13

The additive constant [or “system constant” or “zero correction”],  $z_0$ , is a correction that

is applied to the output of an EODMI,  $s = s' + z_0$ , to account for the offset between the electronic and mechanical centres of an instrument and reflector combination. The magnitude of  $z_0$  can be as high as 35 mm to 90 mm depending on the reflector mounting and EODMI/reflector combination.

- Explain how  $z_0$  can be uniquely determined, with the least number of distance observables, independent of any other information.
- If each distance involved in the unique determination of  $z_0$  is  $\pm 0.002$  m, what is the consequent random uncertainty in  $z_0$ ?
- If the same EODMI as in part b is used elsewhere, say  $s_i' \pm 0.002$  m, what is the random uncertainty in the corrected distance,  $s_i$  [with the value of  $z_0$  being applied]?
- Normally corrections are expected to not significantly contribute to the uncertainty of the quantity that they are correcting. In what way could the random uncertainty in  $z_0$  be improved?
- What type of error contaminates an uncorrected distance,  $s'$ , if  $z_0$  is not applied? Explain how its effect might be misinterpreted if all of the distances, in a traverse between control points, were about the same length.  
Refer to ch. 5 & Example 5.12 [Ogundare, 2016]

#### Study Question 14

The maximum allowable angular misclosure in a traverse of  $n_\beta$  angles is stated as  $M_\beta$  at 99%.

- By showing the propagation of variance, determine the standard deviation,  $\sigma_\beta$ , of each of the  $n_\beta$  angles, considering that each would contribute equally to the actual misclosure  $m_\beta$ . If  $M_\beta$  were  $40''$ , what would be the value of  $\sigma_\beta$  for a traverse of 10 angles?

ans:  $\pm 4.92''$  for each of the angles, being an average of  $n_s$  sets

- If the average from  $n_s$  sets of an angle has a standard deviation of  $\pm \sigma_\beta$ , determine the discrepancy,  $\delta_s$ , between individual sets that would be used as a quality check at the time of observation. If  $\sigma_\beta$  were  $\pm 4.9''$ , what would be the value of the discrepancy if 3 sets were to be observed?

ans:  $|\beta_j - \beta_i| \leq 12.0''$  see Study Question 7  
Refer to ch. 7 [Ogundare, 2016]

#### Study Question 15

Last July, a crew laid out a 1500 m distance from one survey marker to set a second

survey marker. Even though the temperature was +30 °C, they did not apply a meteorological correction but simply used the display value of “1500.000”. You have just measured between the two markers and the uncorrected display, using the same instrument [ $\pm 5$  mm and  $\pm 5$  ppm;  $n_0 = 1.0002936$ , design refractive index number or design refractivity: 278.367 at 15 °C and 760 mmHg] and reflector, is “1500.077” with an ambient temperature of -30 °C.

Determine whether there is a significant difference between the separation of the markers now compared to last July, assuming standard pressure.

Ans: Layout distance was really 1500.0206 m, i.e., 1500.021 m. Measured distance was really 1499.9997 m, i.e., 1500.000 m. Either has a standard deviation of  $\pm 0.009$  m, following  $\sigma_s^2 = [a^2 + b^2 s^2]$ . So, the difference is significant, marginally at 90%.

[Ogundare, 2016]

### Study Question 16

A distance of 1500 m is to be measured. One EODMI,  $\pm 3$  mm and  $\pm 2$  ppm, can measure the overall distance. Another,  $\pm 2$  mm and  $\pm 2$  ppm, would have to measure 500 m at a time. Explain which would be the better choice, considering random and systematic errors, to measure the distance and why.

ans: overall (single distance) would be  $\pm 4.2$  mm; three portions together would be  $\pm 3.9$  mm; consideration of centering error in 3 setups rather than in 1 setup, fidelity of meteorological correction over 500 m rather than 1500 m, application of additive constant three times rather than once.

Refer to ch. 5 [Ogundare, 2016]

### Study Question 17

The ratio of misclosure [“RoM”] in a traverse is often called the “precision” of the traverse. By addressing what contributes to the uncertainty associated with the RoM in a traverse joining two pairs of coordinated [not “control”] monuments, explain whether using the word “precision” is correct and, if not, what would be a better term and why.

ans: consider that the coordinates of the monuments have both random and systematic uncertainty as does the traversing connection.

[Ghilani & Wolf, 2015, Ogundare, 2016]

### Study Question 18

Usually, the purpose of a geodetic network [directions, zenith angles, spatial distances]

is to determine the positions of the stations involved. However, if a network includes points on a sensitive structure and is repeatedly re-observed, the data can be used to monitor the geometric deformation of the structure. And, usually then, the “absolute”, rather than “relative”, movement of the structure points [the “object points”] can be described with respect to the network stations [the “reference points”]. The relative movement of points within the structure usually involved geotechnical instrumentation.

a) Explain the concerns that should be regarded when dealing with “absolute” monitoring over the long term [annually for decades] and what is normally done about these concerns.

b) Explain why geotechnical instrumentation is more likely to be used to monitor relative movement and provide an example to substantiate your explanation.

[Ogundare, 2016]

### Study Question 19

Several federal, provincial or state agencies require, often through statute, the testing or calibration, or both, of equipment used in cadastral surveys. With reference to an example [particular equipment, agency], explain why there is this requirement, what must be tested, what is observed, how the observations are processed [what is done and by whom], what verification of results is to be done, and what use is made of the results.

[Ogundare, 2016]

### Study Question 20

Points A, B, C, D, and E are in a practically straight line. Points A and B have known coordinates and can be considered as errorless. Point E is to be coordinated off points A and B through a traverse having points C and D as intermediate stations. Each point is approximately 200 m from its immediate neighbour. The included angle at B, C, or D is  $\sim 180^\circ$  and the line of the five points can be considered as parallel to the x coordinate axis.

a) If each of the included angles has a standard deviation of  $\pm 5''$ , what is the lateral random error [i.e.,  $\sigma_y$ ] associated with the position of point E?

ans:  $\pm 0.0181$  m

b) What equipment and procedures would you recommend to achieve a standard deviation of  $\pm 5''$  in an angle?

ans: consider the choice of instrument, the effects of centering, levelling, pointing, and reading with regard for the lengths of sight, and the number of sets. Since an appreciable inclination was not mentioned, the effect of levelling can be considered as negligible but such an assumption should be stated in the answer.

c) If azimuths, rather than included angles, were observed [ $\pm 5''$ ] at points B, C, and D, what would be the random lateral error in the position of point E?

ans:  $\pm 0.0083$  m

- d) What equipment and procedures would you recommend to achieve a standard deviation of  $\pm 5''$  in an azimuth?

ans: as in part b, with a limited choice [consider observing in both directions, as well as repeated sets, and taking the average and that the GAK1, or its equivalent, can be mounted on either a T16 or a T2 or another theodolite or total station with the proper support bracket].

- e) Explain what would be the dominant error influence affecting this traverse
- i) in a random manner,
  - ii) in a systematic manner.
- f) Explain why observing azimuths might be preferred over observing included angles in this situation.  
[Ogundare, 2016]

### Study Question 21

Observations on  $\alpha$  *Ursae Minoris*, are done at  $\phi \geq 50^\circ$  in many locations in Canada. As with any other angular measurement, averaging the determinations, by the hour angle method, from several sets improves the precision of the azimuth. The accuracy of the determination would also be improved with several sets. Assuming that an instrument comparable in precision to a Wild T2 [micrometer:  $1''$ ; 28X; plate vial:  $20''/\text{div}$ ; index vial:  $30''$  with coincidence viewing] were being used and that the RO is at least 250 m away, explain what systematic influences would affect a single determination, or set, and how the accuracy would be improved with several sets.

ans: Since  $\phi \geq 50^\circ$ , then  $v \geq 50^\circ$  mislevelment has a significant influence. Direct viewing of the plate vial would propagate into an uncertainty contribution of  $\pm 4.77''$  into the angle to the RO. Consider what must be done between sets for this to be reduced in both random and systematic terms.  
[Ogundare, 2016]

### Study Question 22

Retracement surveys often involve using coordinated survey markers with their

coordinates serving as evidence. In some provinces, these coordinate values originated with surveys done in the 1970s. At that time, orders of control were used to describe the quality of the coordinates.

- a) Explain the concept behind orders of control and how the orders relate to marker spacing, the quality of their coordinates and their relative precision.
- b) Explain how you would integrate this information with modern positioning efforts, esp. if you were incorporating this in the design of a survey.
- c) Explain whether using coordinated monuments in a province-wide system would be suitable as "control" for a project requiring high relative precision, e.g., 1 in 200 000. [Ogundare, 2016]

### **Study Question 23**

A current parcel retracement has resulted in coordinate values for the parcel corners as derived from connecting to four coordinated survey monuments. The earlier survey has parcel corner coordinates resulting from connecting to coordinated survey monuments that are not the same as those used in the current survey. The two sets of parcel corner coordinates are not the same.

- a) Explain why they would be different, having regard for both random and systematic errors.
- b) Explain how you would determine whether the coordinate differences are statistically significant. [Ogundare, 2016]

### **Study Question 24**

A Canada-wide system of elevations is provided by "Vertical Control" benchmarks established by the Geodetic Survey of Canada.

- a) Explain the concept behind orders of vertical control and how the orders relate to benchmark spacing, the quality of their elevations.
- b) Explain how you would integrate this information with modern positioning efforts, esp. if you were incorporating this in the design of a three dimensional survey, e.g., by total station.
- c) Explain whether using these elevations would be suitable as "control" for a project requiring high precision, e.g.,  $\pm 1$  mm or better over an area of 200 m by 200 m. [Ogundare, 2016; Surveys and Mapping Branch, 1978]

### **Study Question 25**

A repetition instrument [theodolite or total station] can be used as a direction instrument

if its lower motion remains clamped. Even so, a crusty older party chief insists that the repetition method is better than the direction method since it is faster in observing and is more precise. Consequently, he has decided to use the repetition method with the instrument even though the specifications say that the angles are to be measured as directions. Explain whether he is justified in doing so.

ans: When only two directions are involved, i.e., in an angle, the repetition method is faster since fewer readings are taken and has a lower standard deviation. If three or more directions are involved, it is more efficient to observe as directions rather than as several angles.

[Ogundare, 2016]

### Study Question 26

#### Corrections to Example 7.4, p.233 (Ogundare [2016])

This example refers to an optical-mechanical theodolite rather than a total station as mistakenly stated in the Example. At the end of the initial design:  $M = 21.9\times$ ; Scale division = 1.1"; centering error = 0.41 mm with re-centering between 2 sets; and sensitivity of plate bubble = 92"/2 mm.

$\sigma_{\alpha}$  in Eqn. (7.53) and Eqn. (7.54) should be replaced with  $\frac{\sigma_{\alpha} \sqrt{n}}{206265}$

Since there is re-centering between sets, forced centering with target and instrument interchange on tripods will not be appropriate (they cannot be done at the same time); the only option for centering is optical plummet, which is not satisfied by the initial design with two sets. Use this initial design to choose a suitable optical-mechanical theodolite that satisfies the initial design, such as Wild T2 ( $M = 28\times$  and  $\text{div} = 1''$ ) with sensitivity of bubble as 20"/div. The remaining concern is how to practically satisfy the centering requirement (Eqn. (4.41)) with optical plummet. Work backward using the chosen theodolite above and try 4 sets as follows:

- With sensitivity of 20"/div and  $\sigma_v = 4''$ , using Eqn. (4.40) & Eqn. (7.57),  $\sigma_{\alpha L} = 0.30''$
- With  $M = 28\times$  and from Eqn. (4.22) & Eqn. (7.49),  $\sigma_{\phi} = 1.07''$
- With  $\text{div} = 1''$  and from Eqn. (4.31) & Eqn. (7.51),  $\sigma_{\theta} = 1.25''$

Substitute the above into Eqn. (7.48) and solve for  $\sigma_{\alpha} = 3.5''$ ; from Eqns. (7.54) – (7.55), the centering error expected,  $\sigma = 1.04$  mm (with re-centering between 4 sets) which satisfies the centering requirement of optical plummet (0.75 mm). On the basis of the above, use Wild T2 ( $M = 28\times$ ,  $\text{div} = 1''$ , sensitivity of bubble = 20"/div, optical plummet and 4 sets of measurements with rotation of the tribrach and re-centering between sets.

Application of Example 7.4 to Total station:

Reading is done electronically by total station, the reading error is negligible compared to the other errors (we can assume  $\sigma_R = 0$  in Eqn. (7.48)) so that  $\sigma_\theta = \sigma\sqrt{3}$  and  $\sigma = 2.24''$

From pointing error in Eqn. (7.50),  $M = 18.9 \times$ ; Eqn. (7.51) does not exist with total station. From centering error in Eqn. (7.55),  $\sigma = 0.47$  mm with centering requirement unsatisfied by re-centering between 2 sets; from leveling error in Eqn. (7.57),  $\sigma_v = 21.1''$ ; sensitivity = 106/div.

**Use initial design to choose instrument:** for example, Leica 702 has standard deviation of a direction measurement in one set =  $2''$  (with standard deviation of the angle in 2 sets as  $2''$ ); electronic leveling dual axis compensator, setting accuracy,  $\sigma_v = 0.5''$ ;  $M = 30 \times$ ; laser centering accuracy (HI = 1.5 m) = 0.75 mm.

Working backward and using the specifications of Leica 702 for 4 sets:

- With sensitivity,  $\sigma_v = 0.5''$ , using Eqn. (4.40) & Eqn. (7.57),  $\sigma_{\theta_L} = 0.04''$
- Pointing error in 4 sets based on the designed error of angle,  $\sigma_{\theta_p} = 2'' \sqrt{2}$  per set,  $\sigma_{\theta_p} = 1.41''$

Substitute the above into Eqn. (7.48) and solve for  $\sigma_{\alpha} = 3.61''$ ; from Eqns. (7.54) – (7.55), the centering error expected (with re-centering between 4 sets),  $\sigma = 1.08$  mm which satisfies the centering requirement of optical plummet (0.75 mm). On the basis of the above, use Leica 702 having standard deviation of a direction measurement in one set as  $2''$ ; electronic leveling dual axis compensator, setting accuracy,  $\sigma_v = 0.5''$ ;  $M = 30 \times$ ; laser centering accuracy (HI = 1.5 m) = 0.75 mm and 4 sets of measurements with rotation of the tribrach and re-centering between sets.