Canadian Board of Examiners for Professional Surveyors
Core Syllabus Item
C 2: LEAST-SQUARES ESTIMATION AND DATA ANALYSIS

Syllabus Topics:

Mathematical modelling; Error propagation and linearization, Concept of adjustment, Least squares adjustment, Variance-covariance propagation, Pre-analysis of survey measurements, Concept of Weights, Normal, Chi-square, t (Student) and F distributions, Confidence intervals, Statistical testing of estimates, residuals and variances, Error ellipses and ellipsoids, General least-squares adjustment, Conditioned, parametric and combined cases, Constraints, Sequential adjustment techniques, Least-squares applications to plane and curvilinear coordinates, Network design, Statistical testing and analysis of estimates, residuals and variances, Error detection, Introduction to least-squares prediction and filtering.

Recommended Prior Knowledge and Skills:

Introductory probability & statistics

Item C1: Mathematics

Courses or extensive experience in plane surveying (horizontal and vertical relative positioning, topographic surveying, familiarity with commonly used surveying instruments, their testing and calibration, procedures, and recording.)

Learning Outcomes:

In order to satisfy the requirements of this syllabus item, candidates should be able to:

1. Apply Knowledge of matrix theory, statistics and estimation:
    - conduct manipulation of matrix algebra involved in adjustment of observations,
    - linearize a non-linear system,
    - apply knowledge of probability and statistics, and
    - demonstrate an understanding of the principles of least square estimation and properties.
2. Analyze measurement errors and modelling, perform random error propagation and pre-analysis of survey measurements:
    - demonstrate an understanding different types of errors and their characteristics,
    - demonstrate an understanding different types of models and characteristics,
    - apply law of random error propagation to determine variance and covariance matrix, and
    - conduct pre-analysis of survey measurements.
3. Formulate least squares adjustment problems (condition, parametric and combined cases):
    - formulate parametric adjustment models (functional and stochastic),
    - formulate condition adjustment models (functional and stochastic), and
    - formulate combined adjustment models (functional and stochastic).
4. Derive adjustment equations of different cases and conduct least square adjustment for geomatics problems such as levelling, traverse, triangulation and trilateration networks:

- derive parametric adjustment equations,
- derive condition adjustment equations,
- derive combined adjustment equations, and
- apply to geomatics problems such as levelling, traverse, triangulation and trilateration networks.

5. Assess the quality of the adjustment solutions (variance factor, variance-covariance matrix, error ellipse):
   - estimate the variance factor,
   - determine variance-covariance matrix of parameters obtained from least square adjustment, and
   - demonstrate an understanding the concept of absolute and relative error ellipse and determine its major axes and orientation.
6. Perform statistical tests on mean and variance to detect and identify outliers in observations (normal, Chi-square, t Student and F distributions, statistical hypotheses, type I and II errors):
   - perform statistical tests on mean and variance to detect and identify outliers in observations,
   - determine the confidence interval of adjusted parameters,
   - select appropriate testing methods (normal, Chi-square, t Student and F distributions), and
   - determine the confidence level and error probability of statistical decisions (significance level, test power, type I and II errors).

Essential Reference Material:

Supplementary Reference Material:

Davis, J.C. [2002]. *Statistical and Data Analysis in Geology*. 3rd, John Wiley & Sons, Toronto, ISBN 0-471-17275-8

This provides a good introduction on matrix algebra and statistical analysis of spatial data

Vanicek, P. & Krakiwsky, E.J. [1986]. *Geodesy: The Concepts*. Part III (Methodology), 2nd, North Holland, New York. ISBN 0 444 87775 4

Part III provides a good treatment of mathematical models, adjustment problem formulation and solutions and assessment of adjustment results.

Wolf, P. R. and Ghilani, C. D. [2006]. *Adjustment Computations: Spatial Data Analysis*. 4th, John Wiley and Sons, Toronto, ISBN 0-471-69728-1.

A good textbook on least squares adjustment and statistical analysis of observations and adjustment results and with many application examples.