Canadian Board of Examiners for Professional Surveyors
Core Syllabus Item
C 1: MATHEMATICS

Study Guide:

1. With respect to functions, continuity and limits:
   - define and describe mathematical functions,
   - define and illustrate continuity of a function at one point, and
   - define and evaluate mathematical limits.

Sample Questions:

Q1.1. What is a mathematical function? Give examples.

Q1.2. Define the continuity of a function f(x) at some point x = xo.

Q1.3. What is a discontinuity of a function f(x) at some point x = xo?

Q1.4. Define the limit for a function f(x) as x approaches xo (i.e. as $x \rightarrow xo$)

Q1.5. Distinguish between the limit from the left and the limit from the right for a function f(x) as x approaches xo on the real line.

2. With respect to differentiation and applications:
   - define differentiability of a function at one point,
   - differentiate simple functions, and
   - interpret the derivatives of a function.

Sample Questions:

Q2.1. What is differentiability of function f(x) at some point x = xo?

Q2.2. Distinguish between continuity and differentiability of f(x) at a point x = xo.

Q2.3. Differentiate simple functions e.g. sin x, cosh x, ex, loge x, log10 x

Q2.4. What are the 2nd, 3rd, … derivatives of the previous functions?

Q2.5. For a function f(x), what is the interpretation of its derivatives $f'(x)$, $f''(x)$,…?

3. With respect to integration, quadratures and applications:
   - define and describe integration of a function,
   - integrate simple functions,
   - describe indefinite and definite integrals, and
   - evaluate numerically definite integrals.

Sample Questions:

Q3.1. What is the indefinite integral of a function f(x)? Give examples.

Q3.2. What are definite integrals of the same function f(x)? Give examples.

Q3.3. Evaluate the integral of $f(x) = \sin x$ between $x = 0$ and $x = \pi/2$.

Q3.4. What is a simple quadrature approximation of the previous integral?

Q3.5. What is a simple error analysis for such a quadrature?

4. With respect to plane curves, tangency and curvature:
   - formulate representations of plane curves,
   - describe the tangent to a curve at one point, and
   - describe the curvature of a curve at one point.

Sample Questions:

Q4.1. What is a plane curve? Give examples.

Q4.2. What are implicit and parametric forms of a simple sinusoid?

Q4.3. What are arclength, slope and curvature for a simple sinusoid?

Q4.4. Given the function $f(x) = \sin x$, what is the direction of tangent at some $x = xo$?

Q4.5. Given the function $f(x) = \sin x$, what is its curvature at some $x = xo$?

5. With respect to sequences, series and Taylor expansions:
   - describe sequences and series,
   - define convergence of sequences and series,
   - formulate tests of convergence for sequences and series, and
   - perform Taylor series expansions of simple functions.

Sample Questions:

Q5.1. What is the difference between a sequence and a series? Give examples.

Q5.2. What is the limit of $1, 1/2, 1/3, \ldots, 1/n, \ldots$ as $n \rightarrow \infty$?

Q5.3. How do verify that $1 + 1/2 + 1/3 + \ldots + 1/n + \ldots$ diverges?

Q5.4. How do verify that $1 + 1/4 + 1/9 + \ldots + 1/n2 + \ldots$ converges?

Q5.5. What is a Taylor expansion of a function $f(x) = \sin x$ at some point $x = xo$?

6. With respect to partial differentiation and differential operators:
   - define and describe partial differentiation,
   - partially differentiate simple functions, and
   - define gradient and Laplacian operators and describe their applications.

Sample Questions:

Q6.1. What is a partial differentiation of some function $f(x, y, z)$? Give examples.

Q6.2. Given $f(x, y, z) = exsin(y+z)$, what are the corresponding partial derivatives?

Q6.3. What is the total derivative of the previous $f(x, y, z)$?

Q6.4. What is $\nabla f(x, y, z)$ for the previous $f(x, y, z)$?

Q6.5. What is $\Delta f(x, y, z) \equiv \nabla 2 f(x, y, z)$ for the previous $f(x, y, z)$?

7. With respect to multiple integrals and numerical approximations:
   - define and describe multiple indefinite and definite integrals, and
   - describe numerical approximation techniques for multiple integrals.

Sample Questions:

Q7.1. What are multiple indefinite and definite integrals of a function? Give examples.

Q7.2. What is the area of the surface $f(x, y) = \sin xy$ inside $0 \leq x \leq \pi/2$ and $0 \leq y \leq \pi/2$?

Q7.3. What is the volume under $f(x, y) = \sin xy$ inside $0 \leq x \leq \pi/2$ and $0 \leq y \leq \pi/2$?

Q7.4. What are some methods for approximating those integrals by summations?

Q7.5. What are quadrature methods for multiple integrals? Give simple examples.

8. With respect to vector operations and analytical geometry:
   - define and describe real and complex vectors,
   - evaluate scalar and vector products of vectors, and
   - express analytical geometry equations or formulae in terms of vectors.

Sample Questions:

Q8.1. What are real and complex vectors? Give examples.

Q8.2. Express real and complex vectors in cartesian and polar coordinates.

Q8.3. What is the inner or scalar product of two arbitrary vectors? Give examples.

Q8.4. What is the cross or vector product of two arbitrary vectors? Give examples.

Q8.5. What is the length of a vector? What is the angle between two vectors?

9. With respect to first and second order linear differential equations and solutions:
   - describe linear ordinary differential equations,
   - describe linear partial differential equations,
   - describe and execute solution methods for simple ordinary differential equations, and
   - describe and execute solution methods for simple partial differential equations

Sample Questions:

Q9.1. What is the difference between ordinary and partial differential equations?

Q9.2. What are the orders in ordinary and partial differential equations? Give examples.

Q9.3. What are the differential equations for $f(x) = \sin x$? $g(x) = \sinh x$? $h(x) = e^x$?

Q9.4. What are the solution methods for the previous differential equations?

Q9.5. Solve the equations $\Delta U(x, y) = 0$ and $\Delta V(x, y, z) = 0$ for Cartesian x, y and z.

10. With respect to introduction to matrix algebra, linear equations and transformations:
    - describe matrices and simple matrix algebra,
    - express the matrix representation of linear algebraic equations and solutions, and
    - express the matrix representation of linear transformations.

Sample Questions:

Q10.1. What is a matrix? What is a column or row vector? Give examples.

Q10.2. Given two matrices A and B, are A + B, AB, A2 + B2, (A + B)2 meaningful?

Q10.3. What is the determinant of a square matrix? What is the matrix inverse?

Q10.4. What are Cramer's rule and Gaussian elimination for linear algebraic systems?

Q10.5. What is the matrix algebra of linear systems of equations? Give examples.

11. With respect to complex variables, linear spaces and subspaces:
    - define and describe complex variables,
    - describe linear real and complex spaces and subspaces, and
    - express the projections in real and complex spaces.

Sample Questions:

Q11.1. What is a real linear space? What is a complex linear space? Give examples.

Q11.2. What are linear dependence and independence of a set of real or complex vectors?

Q11.3. What is a basis for a real or complex linear space? Give simple examples.

Q11.4. What is the dimension of a real or complex linear space? Give simple examples.

Q11.5. What is a subspace of a linear space? What is a projection onto a subspace?

12. With respect to quadratic forms, orthogonal and unitary matrices:
    - define and describe quadratic forms and applications, and
    - define orthogonal and unitary matrices and describe their applications.

Sample Questions:

Q12.1. What is a quadratic form? What is it used for? Give examples.

Q12.2. What are eigenvalues and eigenvectors for a square matrix?

Q12.3. What is a singular value decomposition (SVD) for an arbitrary matrix?

Q12.4. What are orthogonal matrices, e.g. rotation matrices?

Q12.5. What are unitary matrices, e.g. Fourier transform matrices?

13. With respect to spherical geometry and trigonometry:
    - define and describe spherical triangles, and

- explain the methods for the solution of standard spherical triangles and the equations involved and execute those solutions.

Sample Questions:

Q13.1. What are great circles and spherical triangles? Give examples.

Q13.2. What is the spherical law of sines? Give examples of applications.

Q13.3. What is the spherical law of cosines? Give examples of applications.

Q13.4. What is called spherical excess? What is the area of a spherical triangle?

Q13.5. What are right-angled and quadrantal spherical triangles? Napier's rules?