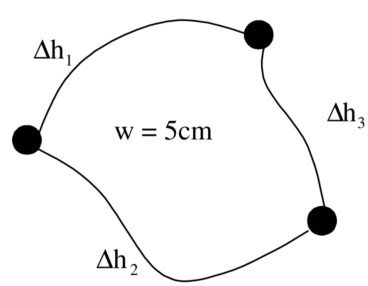
ASSOCIATION OF CANADA LANDS SURVEYORS - BOARD OF EXAMINERS
WESTERN CANADIAN BOARD OF EXAMINERS FOR LAND SURVEYORS
ATLANTIC PROVINCES BOARD OF EXAMINERS FOR LAND SURVEYORS

SCHEDULE I / ITEM 2 **October 2004**

LEAST SQUARES ESTIMATION AND DATA ANALYSIS

Note: This examination consists of _5_ questions on _2_ pages. **Marks**

Time: 3 hours

Q. No		Value	Earned
1	Given the following mathematical models $\mathbf{f}_1(\lambda_1, \mathbf{x}_1) = 0 \quad \mathbf{C}_{\lambda_1} \quad \mathbf{C}_{x_1}$ $\mathbf{f}_2(\lambda_2, \mathbf{x}_1, \mathbf{x}_2) = 0 \quad \mathbf{C}_{\lambda_2} \quad \mathbf{C}_{x_2}$ where f_1 and f_2 are vectors of mathematical models, x_1 and x_2 are vectors of unknown parameter, λ_1 and λ_2 are vectors of observations, $C_{\lambda_1}, C_{\lambda_2}, C_{x_1}$ and C_{x_2} are covariance matrices. a) Formulate the variation function. b) Derive the most expanded form of the least squares normal equation system.	25	
2	Define and explain the following terms: a) Degree of freedom of a linear system b) Accuracy c) Design matrix d) Precision e) Type I and II errors in statistical testing f) Unbiasedness of an estimator	15	
3	Perform a least squares adjustment of the following leveling network in which three height differences Δh_i, i = 1, 2, 3 were observed.  The misclosure w is 5cm. Each Δh_i was measured with a variance of 2 cm^2.	25	

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A baseline of calibrated length (μ) 100.0m is measured 5 times. Each measurement is independent and made with the same precision. The sample mean ($\overline{x}$) and sample standard deviation (s) are calculated from the measurements:

$\overline{x}$ =100.5m s = 0.05m

a) Describe the major steps to test the mean value.

b) Test at the 1% level of confidence if the measured distance is significantly different from the calibrated distance.

The critical value that might be required in the testing is provided in the following table:

Percentiles of t distribution

<table>
<tr><td></td><td colspan="4">t_{α}</td></tr>
<tr><td>Degree of freedom</td><td>$t_{0.90}$</td><td>$t_{0.95}$</td><td>$t_{0.975}$</td><td>$t_{0.99}$</td></tr>
<tr><td>1</td><td>3.08</td><td>6.31</td><td>12.7</td><td>31.8</td></tr>
<tr><td>2</td><td>1.89</td><td>2.92</td><td>4.30</td><td>6.96</td></tr>
<tr><td>3</td><td>1.64</td><td>2.35</td><td>3.18</td><td>4.54</td></tr>
<tr><td>4</td><td>1.53</td><td>2.13</td><td>2.78</td><td>3.75</td></tr>
<tr><td>5</td><td>1.48</td><td>2.01</td><td>2.57</td><td>3.36</td></tr>
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Given the following direct model for y_1 and y_2 as a function of x_1, x_2 and x_3:

$$y_1 = 2x_1 - x_2 - x_3 + 20$$

$$y_2 = x_1 + x_2 - 3x_3 - 50$$

where $x_1 = x_2 = x_3 = 1$ and the covariance matrix of the x's:

$$C_x = \begin{bmatrix} 4 & -2 & -1 \\ -2 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

Compute the covariance matrix $\mathbf{C}_y$ for y's.

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<td>**Total Marks:**</td>
<td>100</td>
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