# CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

## C-1 MATHEMATICS                                    March 2017

**Note:  This examination consists of ten questions on one page.**                    **Marks**

| Q. No | Time: 3 hours | Value | Earned |
|---|---|---|---|
| 1. | a) For a simple function f(x) over the real line, what is the geometrical interpretation of its first derivative f '(x)?  Illustrate the answer graphically. | 5 | |
| | b) For the same simple function f(x) over the real line, what is the geometrical interpretation of its second derivative f ''(x)?  Illustrate the answer graphically. | 5 | |
| 2. | a) Set up the integrals to evaluate the surface area of a unit sphere centred at the origin using spherical polar coordinates. | 5 | |
| | b) Set up the integrals to evaluate the volume of a unit sphere centred at the origin using spherical polar coordinates. | 5 | |
| 3. | a) The expression $(1-x)^{-1}$ can be expanded as the series $1 + x + x^2 + x^3 + \ldots$ only for some values of  real x, i.e. in cases of convergence.  Briefly explain. | 5 | |
| | b) The exponential function $e^x$ can be expanded as  $1 + x + x^2/2 + x^3/6 + \ldots$ for all values of real or even complex x.  Briefly explain the situation. | 5 | |
| 4. | a) Given two planar vectors $\vec{a}$ and $\vec{b}$, what are their dot (or scalar) product in Cartesian and polar coordinates explicitly?  Give simple examples. | 5 | |
| | b) For the same two planar vectors $\vec{a}$ and $\vec{b}$, what are their cross (or vector) product in Cartesian and polar coordinates explicitly?  Give simple examples. | 5 | |
| 5. | a) Let u and v be arbitrary complex numbers. What are the real and imaginary parts of the exponential $e^u$ and logarithm log v explicitly? | 5 | |
| | b) Explain and justify Euler's most famous formula  $e^{\pi\sqrt{-1}} = -1$ | 5 | |
| 6. | a) Given an arbitrary square real matrix, what are its trace and its determinant? | 5 | |
| | b) What is called the singular value decomposition of an arbitrary rectangular matrix?  Give a simple example with explicit dimensions. | 5 | |
| 7. | a) Linear algebraic equations of triangular form are easily solved by forward or backward substitution.  Explain with a simple example of order 3. | 5 | |
| | b) Linear algebraic equations of tridiagonal form are easily solved by Gaussian elimination and forward or backward substitution.  Explain with examples. | 5 | |
| 8. | a) Solve the simple ordinary differential equations  x' = x + 1  and  x' = - x + 2 | 5 | |
| | b) Solve the simple ordinary differential equations  x'' = x + 3  and  x'' = - x + 4 | 5 | |
| 9. | a) Given a second-degree algebraic expression in Cartesian coordinates, what is called its discriminant?  What is it useful for? | 5 | |
| | b) What curves correspond to conic sections?  Illustrate with examples. | 5 | |
| 10. | For two arbitrary locations on the spherical Earth, assumed in one quadrant for simplicity, show how to compute the spherical distance and azimuth in terms of given geocentric latitudes and longitudes.  Illustrate with a simple example. | 10 | |
| | **Total Marks:** | 100 | |