C-1 MATHEMATICS   March 2014

Note: **This examination consists of ten questions on one page.**   **Marks**

Q. No	Time: 3 hours	Value	Earned
1.	a) In Cartesian space, given two points P = (x_1, y_1, z_1) and Q = (x_2, y_2, z_2), what is the algebraic representation of the line PQ?	5	
	b) What is the slope of the preceding line PQ in terms of the Cartesian coordinates of P and Q?	5	
2.	a) Given the Cartesian line element $ds^2 = dx^2 + dy^2$ for the arclength s in the plane, what is the corresponding expression in polar coordinates?	5	
	b) Given the Cartesian line element $ds^2 = dx^2 + dy^2 + dz^2$ for the arclength s in space, what is the corresponding expression in cylindrical coordinates?	5	
3.	a) Considering the series $1 + 1/2 + 1/3 + \ldots + 1/n + \ldots$, does it converge or diverge? Justify your answer.	5	
	b) Considering the series $1 + 1/4 + 1/9 + \ldots + 1/n^2 + \ldots$, does it converge or diverge? Justify your answer	5	
4.	a) What is the gradient ∇ of the function $f(x, y, z) = x^2y^3z^4$ for Cartesian (x,y,z) coordinates?	5	
	b) What is the Laplacian Δ or ∇^2 of the function $f(x, y, z) = x^2y^3z^4$ for Cartesian (x,y,z) coordinates?	5	
5.	a) Considering the complex transformation $z \rightarrow w$ as $w = z + \beta$ for complex β, what are the corresponding Cartesian transformation equations?	5	
	b) Considering the complex transformation $z \rightarrow w$ as $w = \alpha z + \beta$ for complex α and β, what are the corresponding Cartesian transformation equations?	5	
6.	a) For a triangular matrix A, is A^2 also triangular? Illustrate with a small matrix.	5	
	b) For a triangular matrix A, is A^{-1} also triangular? Illustrate with a small matrix.	5	
7.	a) For a rectangular matrix B, what is its singular value decomposition? Illustrate with a simple example.	5	
	b) For a symmetric matrix C, what is its singular value decomposition? Illustrate with a simple example.	5	
8.	a) Set up the integral for the area of a circle of radius r centered at the origin of the coordinate system.	5	
	b) Set up the integral for the area of an ellipse of semi-major axis a and semi-minor axis b centered at the origin of the coordinate system.	5	
9.	a) Given a general polynomial of second degree $a + b x + c x^2$, what are its roots in terms of a, b and c?	5	
	b) Given a general polynomial of third degree $a + b x + c x^2 + d x^3$, are its roots expressible in terms of a, b, c and d? Illustrate using a simple example.	5	
10.	Spherical trigonometry is very useful for spherical Earth computations. What is a spherical triangle? What are the usual geodetic coordinate systems? How are they related to the corresponding usual geocentric Cartesian coordinate system?	10	
	Total Marks:	100	