CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

C-1 MATHEMATICS

October 2011

Note: <u>O. No</u>	This examination consists of ten questions on one page. Time: 3 hours	<u>Mai</u> <u>Value</u>	<u>rks</u> Earned
		5	Larned
1.	a) Define continuity of a function as a limit. Illustrate with a simple diagram.b) Define differentiability of a function as a limit. Illustrate with a simple diagram.	5	
2.	a) Given two points $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$ in Cartesian space, what is	5	
	the vector \overline{PQ} in Cartesian coordinates?		
	b) What is the same vector \overline{PQ} in spherical polar coordinates? Express those vector components in terms of the Cartesian coordinates of P and Q.	5	
3.	a) What is the Taylor series expansion of $e^{\sin x}$ about x=0? Evaluate only the first three terms.	5	
	b) What is the inverse function of $y = e^{\sin x}$?	5	
4.	a) Complex numbers are two-dimensional quantities. Explain briefly with examples.	5	
	b) What are called the n-th roots of unity? Illustrate with $n = 1, 2, 3$.	5	
5.	a) What is the result of the dot or scalar product of two nonzero vectors in three- dimensional space? Give some simple numerical examples.	5	
	b) What is the result of the cross or vector product of two nonzero vectors in three- dimensional space? Give some simple numerical examples.	5	
6.	a) Given a small 3×3 matrix $\mathbf{A} = (1\ 2\ 3\ ; 4\ 5\ 6\ ; 7\ 8\ 9)$, written row-wise, what are the upper triangular matrix U and lower triangular matrix L such that $\mathbf{A} = \mathbf{U} + \mathbf{L}$? Are L and U unique?	5	
	b) Evaluate explicitly the characteristic polynomial of the preceding matrix A.	5	
7.	a) For variable vectors \mathbf{V} in three-dimensional space, what does $\mathbf{V}^{\mathrm{T}}\mathbf{V} = 1$ represent?	5	
	b) For variable vectors \mathbf{V} in three-dimensional space, what does $\mathbf{V}^{\mathrm{T}}\mathbf{W}\mathbf{V} = 1$ represent for some positive diagonal matrix \mathbf{W} ?	5	
8.	a) Given three linear equations: $x + 2y + 3z = 5$, $x + y - 2z = 8$, $x - 2y + z = 12$, evaluate x, y and z by Gaussian elimination.	5	
	b) Solve the preceding three equations for x, y and z using matrix algebra.	5	
9.	a) Given the differential equation: $du / dx + u = 3$, what is the general solution?	5	
	b) Given another differential equation $d^2v / dx^2 + 3v = 0$, what is the general solution?	5	
10.	a) At a point H on the spherical Earth, what is the direction of the normal at H?	5	
	b) At a point G on the ellipsoidal Earth, what is the direction of the normal at G? Briefly describe the general situation.	5	
	Total Marks:	100	