C-1 MATHEMATICS   **March 2012**

Note: This examination consists of ten questions on one page.   **Marks**

Q. No   Time: 3 hours   Value   Earned

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1.	a) Evaluate the following limit as x goes to zero: $\lim\limits_{x \to 0}\left\{\dfrac{3+7x}{5+8x^2}\right\}$.	5	
	b) Evaluate the following limit as y goes to infinity: $\lim\limits_{y \to \infty}\left\{3+e^{-y+1}\right\}$.	5	
2.	a) Formally, $(1-x)^{-1} = 1 + x + x^2 + \dots$ Is that valid for all real x values?	5	
	b) What can be said about the convergence of the series $1 - x + x^2 - x^3 + \dots$?	5	
3.	a) Differentiate $2\sin(\pi - 2x)$ with respect to x and simplify.	5	
	b) Integrate the same function of x, i.e., $2\sin(\pi - 2x)$ over some interval [a, b]	5	
4.	a) Given an ellipse as $\alpha x^2 + \beta y^2 = 1$ in the Cartesian (x, y) plane, set up the integral for its interior area, assuming the parameters α and β are positive.	5	
	b) For the same ellipse, what is the integral to compute the complete arc length around the ellipse?	5	
5.	a) What is the equation for a circle of unit radius at the origin of the complex z plane?	5	
	b) What is the equation for an ellipse of semi-major axis a and semi-minor axis b at the origin of the complex z plane?	5	
6.	a) Given an arbitrary curve y = f(x) in the Cartesian plane, what is the direction of its tangent at some point $y_o = f(x_o)$? Illustrate with a simple example.	5	
	b) For the same curve y = f(x) in the Cartesian plane, what is the direction of its normal at the point $y_o = f(x_o)$? Illustrate with a simple example.	5	
7.	a) What are eigenvalues and eigenvectors for a square matrix? Illustrate with a small matrix of order 3.	5	
	b) What happens when the previous matrix is singular?	5	
8.	a) A linear algebraic system of equations is transformed into an upper triangular system by Gaussian elimination. Explain and illustrate with a simple system of three equations.	5	
	b) Briefly show how to solve an upper triangular system of linear equations and illustrate with the preceding example following the Gaussian elimination.	5	
9.	a) What is Laplace partial differential equation in Cartesian coordinates (x, y) ?	5	
	b) What is Laplace partial differential equation in polar coordinates (r, θ) ?	5	
10.	a) Given two arbitrary points $P_1 = (\varphi_1, \lambda_1)$ and $P_2 = (\varphi_2, \lambda_2)$ on the spherical Earth with latitude φ and longitude λ, derive a formula for the azimuth of the direction P_1P_2 ?	5	
	b) Are there any ambiguities in the formula for the preceding azimuth?	5	
	Total Marks:	100	