

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct. Note: This examination consists of ten questions on one page.

<u>Q. No</u>	<u>Time: 3 hours</u>	<u>Value</u>	<u>Marks</u> <u>Earned</u>
1.a)	Expand $\sin(x+iy)$ and $\cos(x+iy)$ for real x and y with the imaginary i , that is the square-root of -1 .	5	
b)	Verify that $\sin^2 z + \cos^2 z = 1$ for complex $z = x+iy$ with real x, y and imaginary i .	5	
2.a)	Differentiate $y = 2 \cos(3x-\pi)$ with respect to x and simplify.	5	
b)	For an arbitrary function $f(x)$, what is the difference between its continuity and its differentiability at some point $x = x_0$? Illustrate the answer with diagrams.	5	
3.a)	What is the Cartesian equation of a triaxial ellipsoid with semi-axes a, b, c and centre at the origin of the coordinate system?	5	
b)	What is the spherical polar equation of a triaxial ellipsoid with semi-axes a, b, c and centre at the origin of the coordinate system?	5	
4.a)	Evaluate the sum of the arithmetic sequence $\{1, 2, 3, \dots, 98, 99\}$	5	
b)	Evaluate the sum of the geometric sequence $\{1, 2, 4, 8, \dots, 1024\}$	5	
5.a)	For complex variables z and w , what is the nature of the transformation $w = \alpha z$ for some complex scalar α ?	5	
b)	For complex variables z and w , what is the nature of the transformation $w = \alpha z + \beta$ for some complex scalars α and β ?	5	
6.a)	What are the inner and outer products of the two vectors $(1, 3, 5)^T$ and $(7, 9, 8)^T$?	5	
b)	What is the angle between the two preceding vectors?	5	
7.a)	Given a small matrix A with elements $a_{ij} = (i+j)^2$ for $i=1,2,3$ and $j=1,2,3$, what is the determinant of A ?	5	
b)	What is the characteristic polynomial for the preceding matrix A ?	5	
8.a)	Given the following equations: $2x + 3y + 4z = 16$, $3x - 4y + 5z = 6$, and $4x - 5y + 6z = 8$, what are x, y and z by Cramer's rule?	5	
b)	Given the following equations: $2x + 3y + 4z = 0$, $3x - 4y + 5z = 0$, and $4x - 5y + 6z = 0$, what are the possible x, y and z ?	5	
9.a)	What is the area under the curve $y = e^{5x}$ between $x = 0$ and $x = 1$?	5	
b)	What is the length of the curve $y = e^{5x}$ between $x = 0$ and $x = 1$?	5	
10.a)	Given two arbitrary points (ϕ_1, λ_1) and (ϕ_2, λ_2) on the spherical Earth with latitude ϕ and longitude λ , what is a formula for the (spherical arc) distance between them?	5	
b)	Are there any ambiguities in the preceding answer for the spherical distance?	5	
Total Marks:		100	