CANADIAN BOARD OF EXAMINERS FOR PROFESSIONAL SURVEYORS

SCHEDULE I / ITEM 2 March 2010
LEAST SQUARES ESTIMATION & DATA ANALYSIS

Although programmable calculators may be used, candidates must show all formulae used, the substitution of values into them, and any intermediate values to 2 more significant figures than warranted for the answer. Otherwise, full marks may not be awarded even though the answer is numerically correct.

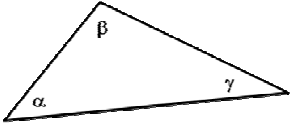**Note: This examination consists of 8 questions on 3 pages.**

Q. No	Time: 3 hours	Value	Earned
1.	Define and explain briefly the following terms: a) Precision and accuracy b) Type I and Type II errors in statistical testing c) Statistically independent and uncorrelated d) Standard deviation and root mean square error	2.5 2.5 2.5 2.5	
2.	Given the following mathematical model $$f(\ell, x) = 0 \quad C_\ell \quad C_x$$ where f is the vector of mathematical models, x is the vector of unknown parameters and C_x is its variance matrix, ℓ is the vector of observations and C_ℓ is its variance matrix. a) Linearize the mathematical model b) Formulate the variation function c) Derive the least squares normal equations d) Derive the least squares solution of the unknown parameters.	3 3 7 7	
3.	The distance between Point A and Point B has been independently measured 5 times with the same precision using a distance measuring device and the standard deviation of the obtained mean distance is 1.58cm. Determine the precision of a single distance measurement. A ←————————————→ B	5	
4.	Given the angle measurements of a triangle along with their standard		

deviations:

Angle	Measurement	Standard Deviation
α	104°38'56"	6.7"
β	33°17'35"	9.9"
γ	42°03'14"	4.3"

Perform least squares adjustment to the problem using

    a) Conditional equations (conditional adjustment)
    b) Observation equations (parametric adjustment)

12.5
12.5

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5. Given the variance-covariance matrix of the horizontal coordinates (x, y) of a survey station, determine the semi-major, semi-minor axis and the orientation of the standard error ellipse associated with this station.

$$C_x = \begin{bmatrix} 0.0484 & 0.0246 \\ 0.0246 & 0.0196 \end{bmatrix} \ m^2$$

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6. A distance has been independently measured 4 times and its sample unit variance obtained from the adjustment $\hat{\sigma}_0^2$ is equal to 1.44 cm. If the a-priori standard deviation σ_0 is 1.0 cm, conduct a statistical test to decide if the adjustment result is acceptable with a significance level of $\alpha = 5\%$. The critical values that might be required in the testing are provided in the following table:

α	0.001	0.01	0.025	0.05	0.10
$\chi^2_{\alpha,\ \upsilon=3}$	16.26	11.34	9.35	7.82	6.25

where $\chi^2_{\alpha,\ \upsilon=3}$ is determined by the equation $\alpha = \int_{\chi^2_{\alpha,\ \upsilon=3}}^{\infty} \chi^2(x)dx$ and υ is the degrees of freedom.

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	An angle has been measured independently 5 times with the same precision and the observed values are given in the following table. Test at the 95% level of confidence if the sample mean is significantly different from the true angle value $45^{o}00'00''$.		

α_1	α_2	α_3	α_4	α_5
$45^{o}00'05''$	$45^{o}00'10''$	$44^{o}59'58''$	$45^{o}00'07''$	$44^{o}59'54''$

The critical value that might be required in the testing is provided in the following table:

Degrees of freedom	t_{α}			
	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$
1	3.08	6.31	12.7	31.8
2	1.89	2.92	4.30	6.96
3	1.64	2.35	3.18	4.54
4	1.53	2.13	2.78	3.75
5	1.48	2.01	2.57	3.36

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8. Given the variance-covariance matrix of the measurement vector $\ell = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$:

$$C_{\ell} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

and the function $x = \ell_1 + \ell_2$, determine C_x.

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Total Marks: 100